Revision Exercise on Probability

1 IJC//II/8

In a game, three containers have the following contents.

A jar contains 10 red balls and 15 blue balls.
A white box contains 4 mint chocolates and 8 almond chocolates.
A black box contains 7 milk chocolates and 3 almond chocolates.

One ball is drawn at random from the jar. If the ball is red, a chocolate will be drawn at random from the white box. If the ball is blue, a chocolate will be drawn at random from the black box. Events X and Y are defined as follows.

X : A red ball is drawn from the jar.

Y : The second item drawn is an almond chocolate.

Find, giving each of your answers as an exact fraction in its lowest terms,

(i) P (Y), [2]

(ii) P ($X \cup Y$), [2]

(iii) P ($X \mid Y$). [2]

Determine whether X and Y are independent events. [2]

2 JJC/II/8

A deck of 36 cards has four colours, red, blue, green and yellow. Each colour set consists of 9 cards numbered 1, 2, 3, …, 9.

(a) One card is taken at random from the deck. Events A and B are defined as follows:

A: The card taken is blue.

B: The card taken is a 1.

(i) Determine whether A and B are independent events. [2]

(ii) Describe in words what the event $A \cup B$ represents and find the probability of this event. [2]

(iii) Find $\mathrm{P}(A' \mid B')$. [2]

(b) Three cards are taken from the deck, at random and without replacement. Find the probability that all three cards are of different colours with the same number. [2]

(c) James takes 20 cards from this deck of 36 cards at random and without replacement. Find the probability that none of these cards are numbered '1'. [2]

3 MI/II/11

(a) Two fair dice are thrown simultaneously. Events A, B and C are defined as follow:

A: the total score is an even number,
B: at least one six appears,
C: the total score is ten.

(i) Find P(A), P(B) and P(C). [3]
(ii) Determine whether B and C are independent. [2]
(iii) Given that A has occurred, find the probability of C not happening. [2]

(b) Three friends Daisy, Elvin and Flora share a flat. Each evening there is only one incoming call. The probability of an incoming telephone call in the evening being for Daisy, Elvin and Flora is 0.2, 0.3 and 0.5 respectively. The probability that Daisy is at home is 0.75. The corresponding probabilities for Elvin and Flora are 0.5 and 0.8 respectively. All the probabilities are independent.

Find the probability that

(i) the call is for Daisy and Daisy is at home, [2]
(ii) the person to whom the call is made is at home. [3]

4 MJC/II/9

Cards are drawn at random and without replacement from a deck of 20 cards, which are numbered $1, 2, 3, \ldots, 20$.

(a) Find the probability that of three cards drawn from the deck,
(i) all three cards are even; [1]
(ii) exactly one of the three cards is even. [2]

(b) Let
A be the event that the first card drawn is less than or equal to 5;
B be the event that the second card drawn is greater than or equal to 5.

Find, in any order,

(i) $\mathrm{P}(B)$, [2]

(ii) $\mathrm{P}(A \cap B)$, [2]

(iii) $\mathrm{P}(B|A)$. [1]

5 RJC/II/6

(a) Two events A and B are such that $\mathrm{P}(A) = 0.2$ and $\mathrm{P}(A \cup B) = 0.6$.

Find $\mathrm{P}(B)$ if

(i) A and B are mutually exclusive, **[1]**

(ii) A and B are independent. **[2]**

(b) In 2008, an insurance company classifies 10% of their car policy holders as "good risks", 60% as "average risks" and 30% as "bad risks". Their statistical database in 2008 has shown that of those classified as "good risks", only 1% were involved in at least one accident whereas of those classified as "bad risks", 25% were involved in at least one accident. As for those classified as "average risks", 15% were involved in at least one accident. nd the probability that

(i) a randomly chosen car policy holder was involved in at least one accident in 2008. **[2]**

(ii) a randomly chosen car policy holder was classified as "good risks" if the car policy holder was not involved in any accidents in 2008. **[3]**

6 SRJC/II/7

A box contains 20 balls that are labeled from 1 to 20. Raju and Kelvin take turns to pick a ball from the box with Kelvin drawing a ball first. The game will stop when a prime number (2, 3, 5, 7, 11, 13, 17, 19) is drawn.

(a) If the balls are drawn **without replacement**, find the probability that Kelvin is the first to pick a prime number given that the game ends on the second draw of the player. [3]

(b) If the balls are now drawn **with replacement**, find the probability that Raju is the first to pick a prime number. [3]

7 TPJC/II/8

(a) The events A and B are such that

$$\mathrm{P}(A) = \frac{1}{2}, \qquad \mathrm{P}(A \mid B') = \frac{2}{3}, \qquad \mathrm{P}(A \mid B) = \frac{2}{7}$$

where B' is the event "B does not occur"

Find P(B) and state with reason, whether A and B are independent. [5]

(b) A container contains 15 balls numbered 1 to 15. Balls are drawn successively from the container, one at a time with replacement.
A "strike" is obtained when the first number is drawn again.

(i) Calculate the probability that the first "strike" is obtained on or before the 4^{th} draw. [2]

(ii) Given that the first "strike" occurs on the 3^{rd} draw, find the probability that the second "strike" is obtained on or before the 7th draw. [2]

8 VJC/II/6

In a quiz show, questions are generated randomly. The probability that a contestant gets an easy question is 0.6 while the probability of getting a challenging question is 0.4. The probability of any contestant giving the correct answer to an easy question is 0.8 while the probability of giving the correct answer to a challenging question is 0.3.

Find the probability that

(i) a contestant gets 3 easy questions in a row, [1]

(ii) a contestant gives the correct answer twice in a row, [4]

(iii) a contestant got a challenging question given that the contestant gave the correct answer. [3]

Answers

1 (i) $\left(\frac{67}{150}\right)$; (ii) $\frac{29}{50}$; (iii) $\frac{40}{67}$

2 (a) (ii) $\frac{1}{3}$; (iii) $\frac{3}{4}$; (b) $\frac{3}{595}$; (c) 0.0309

3 (a) (i) 1/12; (iii) 5/6 ; (b) (i) 0.15; (ii) 0.70

4 (i) $\frac{2}{19}$; (ii) $\frac{15}{38}$; (b) (i) $\frac{4}{5}$ (ii) $\frac{79}{380}$ (iii) $\frac{79}{95}$

5 (a) (i) 0.4 (ii) 0.5 (b) (i) 0.166 (ii) 0.119

6 (i) 17/27 (ii) 3/8

7 (a) $\frac{7}{16}$, no; (b) (i) $\frac{631}{3375}$; (ii) $\frac{12209}{50625}$

8 (i) 0.216; (ii) 0.36; (iii) 0.2