Summary on Probability

Definition: $\mathrm{P}(A) = \dfrac{\text{no. of elements in } A}{\text{no. of elements in S}} =$ e.g.

Laws: 1) $\_\_\_ \le \mathrm{P}(A) \le \_\_$

2) $\mathrm{P}(A') =$ e.g.

3) $\mathrm{P}(A \cup B) =$ e.g.

Conditional Probability: $\mathrm{P}(A \mid B) =$

$=$ (reduced sample space to B)

e.g.

Mutually Exclusive and Independent Events:

Events

Cannot occur simultaneously → **Mutually Exclusive Events**

$\mathrm{P}(A \cap B) =$

$\Rightarrow \mathrm{P}(A \cup B) =$

Occurrence does not influence each other → **Independent Events**

$\mathrm{P}(A \cap B) =$

or $\mathrm{P}(A|B) =$

or $\mathrm{P}(B|A) =$

Note: (1) If A and B are both independent and mutually exclusive, then $\mathrm{P}(A) = \_\_$ or $\mathrm{P}(B) = \_\_$.
(2) If A and B are independent events, then A' and B' are _______________ events.

Methods: In solving probability problems, we can use the following methods:

(I) **List or Table of Outcomes** -- when sample space is not too ______ so that all possible outcomes can be _______ or ____________.

Example 1 Two fair dice are thrown. Events A, B *and* C are defined by

A : The first die shows 5.
B : The total score is 7.
C : The total score is 9.

(i) Determine whether A and B are independent
(ii) Determine whether A and C are independent

Solution

(II) **Venn Diagram** -- when combinations of events such as _______, ________, … , are involved.

Eg. Given that P(A) = 0.3, P(B) =0.4 and P($A \cap B$) = 0.1. Find

(i) P($(A \cup B)'$) (ii) P($A' \cap B$) (iii) P($A \cup B'$)

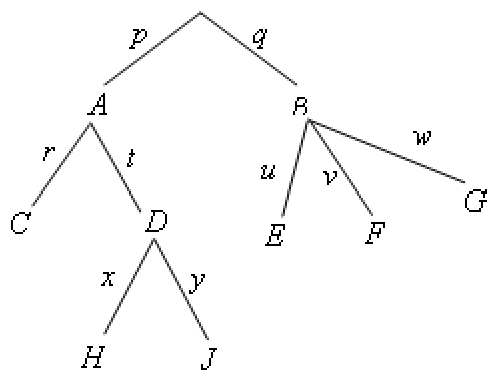
(III) **Permutations and Combinations --** when sample space is ______ and the problem involved permutations and combinations.

Eg. A class consists of 8 boys and 7 girls. Four students are chosen at random to take part in a maths quiz. Find the probability that

(i) exactly 2 girls are chosen,
(ii) all 4 chosen are girls,
(iii) at least 1 boy is chosen.

(IV) **Probability Tree Diagram** -- when the problem involved sequences of events and each sequence has only a ______ possible outcomes.

The **root** of the tree is usually left blank but is convenient to think of it as representing the ______ _______.
Each **node**, such as A, B, C, etc represents an _______.
The **number** indicated on each branch represents the ___________ probability of the event at the end node given that all the events at the previous nodes have occurred.



$P(A) =$ ___ and $P(B) =$ ___

$P(C \mid A) =$ ___and $P(E \mid B) =$ ___

$P(C \cap A) =$
$=$

$p + q =$ ___, $r + t =$ ___, $u + v + w =$___.

Eg. A bag contains 4 red and 6 black balls. One ball is drawn at random. If it is black, it is replaced in the bag ; but if it is red, it is not replaced. A second ball is then drawn.
Let R_1 denotes the event "the first ball is red" and R_2 denotes the event "the second ball is red". Find (i) $P(R_1)$ (ii) $P(R_2 \mid R_1)$
(iii) $P(R_2)$ (iv) $P(R_1 \mid R_2)$ (v) $P(R_1 \cup R_2) - P(R_1 \cap R_2)$