Differentiate $\sin\left(\frac{1}{3}x^{\circ}\right)$ with respect to x.

Solution

$$\frac{d}{dx}\left(\sin \tfrac{1}{3}x^{\circ}\right) = \frac{d}{dx}\left(\sin \tfrac{1}{3}x \times \tfrac{\pi}{180}\right)$$
$$= \frac{d}{dx}\left(\sin \tfrac{\pi}{540}x\right)$$
$$= \frac{\pi}{540}\cos\frac{\pi}{540}x = \frac{\pi}{540}\cos\tfrac{1}{3}x^{\circ}$$

Differentiate $4^{\cot 3x}$ with respect to x.

Solution

$$\frac{d}{dx}\left(4^{\cot 3x}\right) = 4^{\cot 3x}\left(-\operatorname{cosec}^2 3x\right)(3)\ln 4$$
$$= -3\ln 4\left(\operatorname{cosec}^2 3x\right)\left(4^{\cot 3x}\right)$$

Differentiate $\log_2 x^5$ with respect to x.

Solution

$$\frac{d}{dx}\left(\log_2 x^5\right) = \frac{d}{dx}\left[5\frac{\ln x}{\ln 2}\right]$$
$$= \frac{5}{\ln 2}\left(\frac{1}{x}\right) = \frac{5}{x\ln 2}$$

Differentiate $(\sin x)^x$ with respect to x.

Solution

Let $y = (\sin x)^x$

Taking ln on both sides,

$$\ln y = x\ln(\sin x)$$

Differentiating wrt x:

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{\sin x}\cdot\cos x\right) + \ln(\sin x)$$
$$= x\cot x + \ln(\sin x)$$
$$\therefore \frac{dy}{dx} = y\left[x\cot x + \ln(\sin x)\right]$$

Find $\dfrac{d^2y}{dx^2}$ if $\tan y = x^2$

Solution

$$\tan y = x^2$$

Diff. wrt. x :

$$\sec^2 y \frac{dy}{dx} = 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{\sec^2 y} = \frac{2x}{1+\tan^2 y} = \frac{2x}{1+x^4}$$

Diff. wrt x :

$$\frac{d^2y}{dx^2} = \frac{(1+x^4)(2) - 2x(4x^3)}{(1+x^4)^2}$$

$$= \frac{2+2x^4-8x^4}{(1+x^4)^2} = \frac{2-6x^4}{(1+x^4)^2}$$

Find $\dfrac{d^2y}{dx^2}$ if $\dfrac{dy}{dx} = \dfrac{4y^2-3}{x+1}$.

Solution

$$\frac{dy}{dx} = \frac{4y^2-3}{x+1}$$

Diff. wrt. x :

$$\frac{d^2y}{dx^2} = \frac{(x+1)(8y\frac{dy}{dx}) - (4y^3-3)(1)}{(x+1)^2}$$

$$= \frac{(x+1)\left(8y\left(\frac{4y^2-3}{x+1}\right)\right) - (4y^3-3)}{(x+1)^2}$$

$$= \frac{8y(4y^2-3) - (4y^3-3)}{(x+1)^2}$$

$$= \frac{(8y-1)(4y^2-3)}{(x+1)^2}$$

Alternatively,

$$(x+1)\frac{dy}{dx} = 4y^2 - 3$$

$$(x+1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 8y\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{(8y-1)\frac{dy}{dx}}{(x+1)} = \frac{(8y-1)(4y^2-3)}{(x+1)^2}$$