JJC/2010/I/10(ii)

(ii) By sketching the graphs of $y = 3e^x$ and $y = x + 3$, or otherwise, solve the inequality

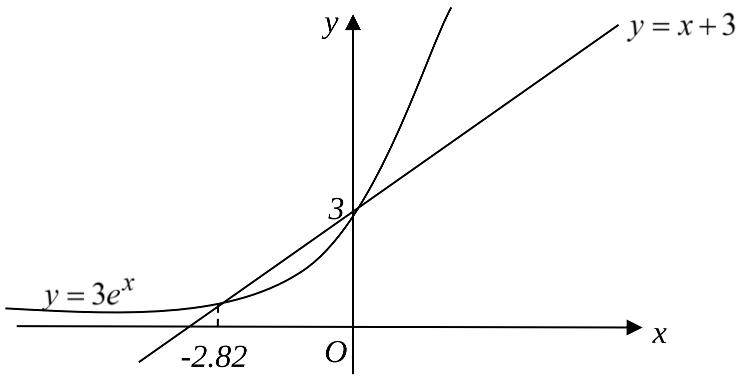
$$3e^x > x + 3.$$ [3]

Hence find

$$\int_{-2}^{2} \left|3e^x - x - 3\right| dx,$$

giving your answer in an exact form. [4]

Solution



From the graphs, $x < -2.82$ or $x > 0$.

For $x < -2.82$ or $x > 0$, $3e^x > x + 3$

i.e. $3e^x - x - 3 > 0$ if $x < -2.82$ or $x > 0$

For $-2.82 < x < 0$, $3e^x < x + 3$

i.e. $3e^x - x - 3 < 0$ if $-2.82 < x < 0$

$\int_{-2}^{2} \left|3e^x - x - 3\right| dx$

$= \int_{-2}^{0} -(3e^x - x - 3)dx + \int_{0}^{2} (3e^x - x - 3)dx$

$= -[3e^x - \frac{x^2}{2} - 3x]_{-2}^{0} + [3e^x - \frac{x^2}{2} - 3x]_{0}^{2}$

$= -[3 - (3e^{-2} - 2 + 6)] + [(3e^2 - 2 - 6) - 3]$

$= 3e^{-2} + 3e^2 - 10$

ACJC/2010/II/1

Find the exact value of $\displaystyle\int_{-1}^{1} \left| e^{2x} - \frac{1}{e^{2(x-1)}} \right| \, dx.$ [4]

Solution

$$\int_{-1}^{1} \left| e^{2x} - \frac{1}{e^{2(x-1)}} \right| \, dx$$

$$= -\int_{-1}^{1/2} e^{2x} - \frac{1}{e^{2(x-1)}} \, dx + \int_{1/2}^{1} e^{2x} - \frac{1}{e^{2(x-1)}} \, dx$$

$$= -\left[\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2(x-1)} \right]_{-1}^{1/2} + \left[\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2(x-1)} \right]_{1/2}^{1}$$

$$= \frac{1}{2}\left(e^4 + e^2 - 4e + e^{-2} + 1\right)$$