RVHS/I/9

It is given that $\mathrm{f}(x)=\ln(1+\sin x)$, where $-\frac{3\pi}{8}\le x\le\frac{3\pi}{8}$.

(i) Show that $(1+\sin x)\mathrm{f}'(x)=\cos x$. [1]

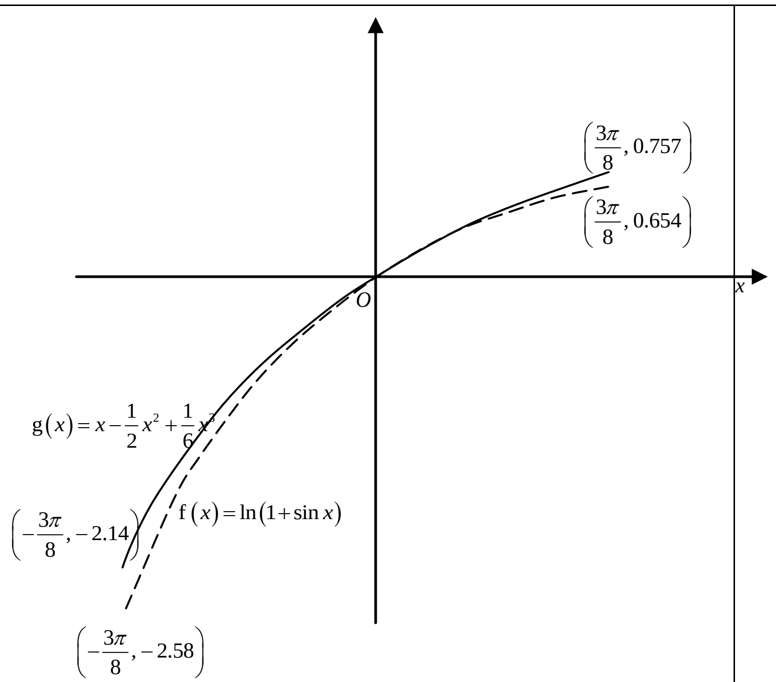
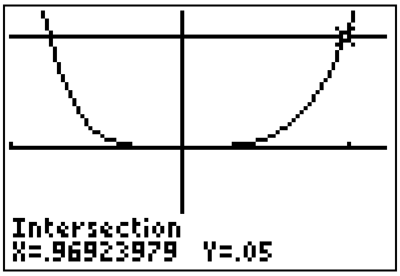
(ii) By further differentiation of this result, find the Maclaurin series for $\mathrm{f}(x)$, up to and including the term in x^3. Hence, write down the equation of the tangent of $y=\mathrm{f}(x)$ at $x=0$. [5]

Denote the Maclaurin series of $\mathrm{f}(x)$ in (ii) by $\mathrm{g}(x)$.

(iii) On the same diagram, sketch the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{g}(x)$ for $-\frac{3\pi}{8}\le x\le\frac{3\pi}{8}$. [2]

(iv) Find, for $-\frac{3\pi}{8}\le x\le\frac{3\pi}{8}$, the set of values of x for which the value of $\mathrm{g}(x)$ is within ± 0.05 of the value of $\mathrm{f}(x)$. [2]

(i)	$\mathrm{f}(x)=\ln(1+\sin x)$ $\therefore \mathrm{f}'(x)=\frac{\cos x}{1+\sin x}$ $\Rightarrow (1+\sin x)\mathrm{f}'(x)=\cos x$ (shown)
(ii)	Differentiate with respect to *x*: $(1+\sin x)\mathrm{f}''(x)+(\cos x)\mathrm{f}'(x)=-\sin x$ Differentiate with respect to *x*: $(1+\sin x)\mathrm{f}'''(x)+(\cos x)\mathrm{f}''(x)-(\sin x)\mathrm{f}'(x)+(\cos x)\mathrm{f}''(x)=-\cos x$ At $x=0$: $\mathrm{f}(x)=0$, $\mathrm{f}'(x)=1$, $\mathrm{f}''(x)=-1$, $\mathrm{f}'''(x)=1$ $\therefore \mathrm{f}(x)\approx x-\frac{x^2}{2}+\frac{x^3}{6}$ Equation of tangent of $y=\mathrm{f}(x)$ at $x=0$ is $y=x$.

(iii)		
(iv)	To find x such that $\lvert f(x) - g(x) \rvert < 0.05$ Using GC, draw the graph of $y = \lvert f(x) - g(x) \rvert$ to find the intersection with $y = 0.05$.  $-0.7745598 < x < 0.96923979$ Range is $-0.774 < x < 0.969$.	

N2008/P2/1

1 Let $f(x) = e^x \sin x$.

(i) Sketch the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3$. [2]

(ii) Find the series expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [3]

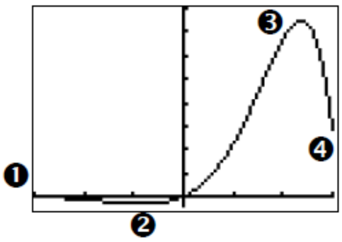
Denote the answer to part **(ii)** by $g(x)$.

(iii) On the same diagram as in part **(i)**, sketch the graph of $y = g(x)$. Label the two graphs clearly. [1]

(iv) Find, for $-3 \leqslant x \leqslant 3$, the set of values of x for which the value of $g(x)$ is within ±0.5 of the value of $f(x)$. [3]

Solution:

(i) [Use the GC to sketch the graph.]



[Highlight the important graph features.]

❶ Left end-point: $(-3, -0.00703)$

❷ Minimum turning point: $(-0.785, -0.322)$

❸ Maximum turning point: $(2.36, 7.46)$

❹ Right end-point: $(3, 2.83)$

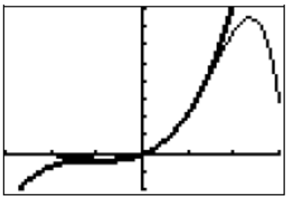
(ii) $f(x) = \left(1 + x + \frac{x^2}{2!} + \ldots\right)\left(x - \frac{x^3}{3!} + \ldots\right)$

[Refer to the expansions of e^x and $\sin x$ in the formulae booklet.]

$= x - \frac{x^3}{6} + x^2 + \frac{x^3}{2} + \ldots$ [Expand.]

$\approx x + x^2 + \frac{1}{3}x^3$ ❖

(iii)



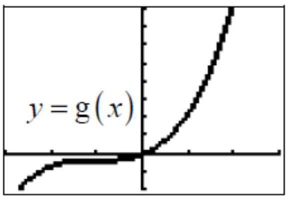
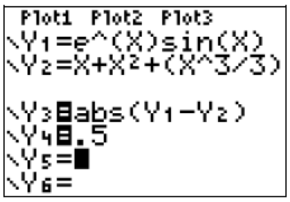
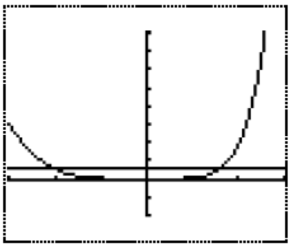


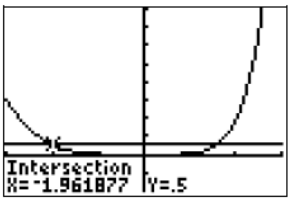
Figure 1 (top) shows both graphs. It is important to draw them in a way that **one is tangent to the other at the origin**.

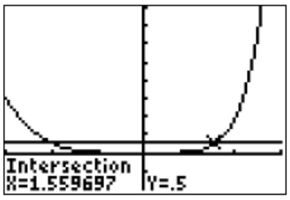
Figure 2 (bottom) shows the shape of $y = \mathrm{g}(x)$ before we draw it with accuracy on the same diagram together with the graph of $y = \mathrm{f}(x)$.

(iv) The value of $\mathrm{g}(x)$ is within ± 0.5 of the value of $\mathrm{f}(x)$ means: $|\mathrm{f}(x) - \mathrm{g}(x)| < 0.5$.









From GC, $-1.96 < x < 1.56$. ❖

[We should give the diagram (in dotted borders) as working.]