| | |
|---|---|
| 1(i) | When x = 0,<br>$2t^2 + t = 0$<br>$t(2t+1) = 0$<br>$t = 0, t = -\frac{1}{2}$<br>The points are $(0,0), (0, 11/8)$<br><br>When y = 0<br>$t^3 - 3t = 0$<br>$t(t^2 - 3) = 0$<br>$t = 0, t = \pm\sqrt{3}$<br>The points are (0,0), ( $6 \pm \sqrt{3}$ ,0) |
| (ii) | $\frac{dx}{dt} = 4t + 1$, $\frac{dy}{dt} = 3t^2 - 3$, $\frac{dy}{dx} = \frac{3t^2 - 3}{4t + 1}$<br><br>Tangent parallel to the y-axis $4t + 1 = 0 \Rightarrow t = \frac{-1}{4}$<br>$(\frac{-1}{8}, \frac{47}{64})$ Equation of tangent $x = -\frac{1}{8}$ |
| (iii) | At $t = -\frac{1}{2}$ $x = 0, y = \frac{11}{8}, \frac{dy}{dx} = \frac{9}{4}$<br>Equation of tangent $y - \frac{11}{8} = \frac{9}{4}x$<br><br>$t^3 - 3t - \frac{11}{8} = \frac{9}{4}\left(2t^2 + t\right)$<br>Solving $t = \frac{11}{2}, -\frac{1}{2}$<br><br>At $t = \frac{11}{2}$, the point is $\left(66, \frac{1199}{8}\right)$ |

2. $y = (x+2)(x-1)(x-2)$

$\frac{dy}{dx} = (x+2)(x-1) + (x-1)(x-2) + (x+2)(x-2)$

$= 3x^2 - 2x - 4$

At stationary pts, $\frac{dy}{dx} = 0$

$3x^2 - 2x - 4 = 0$

$x = \frac{2 \pm \sqrt{4 - 4(3)(-4)}}{2(3)}$

$= \frac{1 \pm \sqrt{13}}{3}$

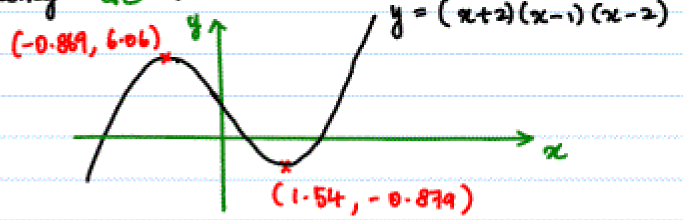
$= 1.54$ or $-0.869$

$\frac{d^2y}{dx^2} = 6x - 2$

At $x = 1.54$, $\frac{d^2y}{dx^2} = 6(1.54) - 2 > 0$

$\therefore (1.54, -0.879)$ is a min pt

Using GC :




By CALC command,

Min pt $= (1.54, -0.879)$

Max pt $= (-0.869, 6.06)$

(i) Curve is strictly ↑ing when $x < -0.869$ or $x > 1.54$

(ii) Curve is strictly ↓ing when $-0.869 < x < 1.54$

(iii) When curve is concave upward,

$\frac{d^2y}{dx^2} = 6x - 2 > 0$

$\therefore x > \frac{1}{3}$

(iv) When curve is concave downward,

$\frac{d^2y}{dx^2} = 6x - 2 < 0$

$\therefore x < \frac{1}{3}$

3(i) Surface area of sphere = $4\pi r^2$

Exposed surface area of cylinder, S $= 2\pi rh + \frac{1}{2}(4\pi r^2)$

$= 2\pi rh + 2\pi r^2$ (shown)

(ii) Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of toy = $\pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \pi r^2 h - \left(\frac{2}{3}\pi r^3\right)$

Since volume of toy is kept constant at $\frac{1}{3}$ m$^3$, we have

$$\pi r^2 \left(h - \frac{2}{3}r\right) = \frac{1}{3} \quad \Rightarrow \quad h = \frac{1}{3\pi r^2} + \frac{2}{3}r \quad \text{----------} \ (*)$$

Sub (*) into S, we have S = $2\pi r\left(\frac{1}{3\pi r^2} + \frac{2}{3}r\right) + 2\pi r^2$

$$= \frac{2}{3r} + \frac{10}{3}\pi r^2$$

and $\quad \frac{dS}{dr} = -\frac{2}{3r^2} + \frac{20}{3}\pi r$

At stationary value, $\quad \frac{dS}{dr} = 0$

$$\Rightarrow \quad \frac{2}{3r^2} = \frac{20}{3}\pi r$$

$$\Rightarrow \quad \pi r^3 = \frac{1}{10}$$

$$\Rightarrow \quad r = \left(\frac{1}{10\pi}\right)^{\frac{1}{3}}$$

$$\frac{d^2S}{dr^2} = \frac{4}{3r^3} + \frac{20}{3}\pi$$

When $r = \left(\frac{1}{10\pi}\right)^{\frac{1}{3}}$, $\frac{d^2S}{dr^2} > 0$

Therefore S is minimum when $r = \left(\frac{1}{10\pi}\right)^{\frac{1}{3}}$ m

4. Let the depth of water be $h$

Volume of water, V $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(h \tan 45^\circ h\right) = \frac{1}{3}\pi h^3$

and $\frac{dv}{dh} = \pi h^2$

After 3 minutes (180 seconds), volume of water running out

= 180 x 2 = 360 $cm^3$

V = initial volume – volume running out

= 390 – 360 = 30 $cm^3$

and thus $\frac{1}{3}\pi h^3 = 30$

$h^3 = \frac{90}{\pi} \qquad \Rightarrow h = \left(\frac{90}{\pi}\right)^{1/3}$

$\frac{dv}{dt} = \frac{dv}{dh} \times \frac{dh}{dt}$

$2 = \pi\left[\left(\frac{90}{\pi}\right)^{1/3}\right]^2 \times \frac{dh}{dt}$

$\therefore \quad \frac{dh}{dt}$ = 0.0680 cm/s