Integration Assignment

1(a) $\int \left(1+\frac{2}{x}\right)^2 dx = \int \left(1+\frac{4}{x}+\frac{4}{x^2}\right) dx$

$= x + 4\ln|x| + 4\left(\frac{x^{-1}}{-1}\right) + c = x + 4\ln|x| - \frac{4}{x} + c$ #

(b) $\int \frac{4}{(4+x^2)\tan^{-1}\left(\frac{x}{2}\right)} dx$ $\qquad \frac{d}{dx}\left(\tan^{-1}\frac{x}{2}\right) = \frac{1}{1+\left(\frac{x}{2}\right)^2}\left(\frac{1}{2}\right)$

$= 2\int \frac{2}{(4+x^2)\tan^{-1}\left(\frac{x}{2}\right)} dx$ $\qquad = \frac{1}{\frac{4+x^2}{4}}\left(\frac{1}{2}\right) = \frac{2}{4+x^2}$

$= 2\ln\left|\tan^{-1}\left(\frac{x}{2}\right)\right| + c$ #

(c) $\int \frac{1}{1+2x-x^2} dx = \int \frac{1}{-(x^2-2x-1)} dx = \int \frac{1}{-[(x-1)^2-1-1]} dx$

$= \int \frac{1}{[(\sqrt{2})^2-(x-1)^2]} dx = \frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+(x-1)}{\sqrt{2}-(x-1)}\right| + c$ #

(d) $\int \frac{2x^3+3x+4}{2x^2+1} dx$

$$\begin{array}{r} x \\ 2x^2+1 \,\big)\, \overline{2x^3+3x+4} \\ -(2x^3+x) \\ \hline 2x+4 \end{array}$$

$= \int x + \frac{2x+4}{2x^2+1} dx$

$= \frac{x^2}{2} + \frac{1}{2}\int \frac{4x}{2x^2+1} dx + 4\int \frac{1}{2x^2+1} dx$

$= \frac{x^2}{2} + \frac{1}{2}\ln(2x^2+1) + 2\int \frac{1}{x^2+\frac{1}{2}} dx$

$= \frac{x^2}{2} + \frac{1}{2}\ln(2x^2+1) + 2\left[\frac{1}{\left(\frac{1}{\sqrt{2}}\right)}\tan^{-1}\frac{x}{\left(\frac{1}{\sqrt{2}}\right)}\right] + c$

$= \frac{x^2}{2} + \frac{1}{2}\ln(2x^2+1) + 2\sqrt{2}\tan^{-1}\sqrt{2}x + c$ #

2. $\frac{d}{dx}(x\cos x) = x(-\sin x) + \cos x = -x\sin x + \cos x$ #

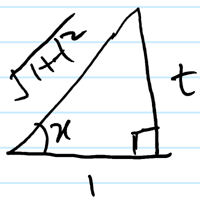
$\int(-x\sin x + \cos x)\, dx = x\cos x + c$

$-\int x\sin x\, dx + \sin x = x\cos x + c$

$\therefore \int x\sin x\, dx = \sin x - x\cos x + c$ #

3. $\int \frac{1}{3+2\sin^2 x} dx$, $t = \tan x \Rightarrow \frac{dt}{dx} = \sec^2 x = 1+t^2$

$= \int \frac{1}{3+2\left(\frac{t}{\sqrt{1+t^2}}\right)^2}\left(\frac{dt}{1+t^2}\right)$



$\sin x = \frac{t}{\sqrt{1+t^2}}$

$= \int \frac{1}{\frac{3(1+t^2)+2t^2}{\cancel{1+t^2}}}\left(\frac{dt}{\cancel{1+t^2}}\right)$

$$= \int \frac{1}{5t^2+3}\,dt = \frac{1}{5}\int \frac{1}{t^2+\frac{3}{5}}\,dt = \frac{1}{5}\left[\frac{1}{\sqrt{\frac{3}{5}}}\tan^{-1}\frac{t}{\left(\sqrt{\frac{3}{5}}\right)}\right] + c$$

$$= \frac{1}{\sqrt{15}}\tan^{-1}\sqrt{\frac{5}{3}}\,t + c = \frac{1}{\sqrt{15}}\tan^{-1}\left(\sqrt{\frac{5}{3}}\tan x\right) + c_{\#}$$

4. $$\int \tan^{-1}(3x)\,dx = \int (1)\tan^{-1}(3x)\,dx$$

$$= [\tan^{-1}(3x)](x) - \int x\left(\frac{1}{1+(3x)^2}(3)\right)\,dx$$

$$= x\tan^{-1}(3x) - \int \frac{3x}{1+9x^2}\,dx$$

$$= x\tan^{-1}(3x) - \frac{1}{6}\int \frac{18x}{1+9x^2}\,dx$$

$$= x\tan^{-1}(3x) - \frac{1}{6}\ln(1+9x^2) + c_{\#}$$