Vectors (I) – Assignment

1.

$$(i)\left|\overrightarrow{AB}\right| = \left|\begin{pmatrix}3\\-5\\4\end{pmatrix}\right| = \sqrt{50} = 5\sqrt{2}$$

$$(ii)\cos B\hat{A}C = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|} = \frac{\begin{pmatrix}3\\-5\\4\end{pmatrix}.\begin{pmatrix}2\\-2\\-1\end{pmatrix}}{5\sqrt{2}\sqrt{9}} = \frac{12}{15\sqrt{2}} \Rightarrow B\hat{A}C = 55.6° = 56°$$

$$(iii)\frac{1}{2}\left|\overrightarrow{BA} \text{ x } \overrightarrow{BC}\right| = \frac{1}{2}\left|\begin{pmatrix}-3\\5\\-4\end{pmatrix} \text{ x } \begin{pmatrix}-1\\3\\-5\end{pmatrix}\right| = \frac{1}{2}\left|\begin{pmatrix}-13\\-11\\-4\end{pmatrix}\right| = \frac{3}{2}\sqrt{34}$$

$$(iv)\left|\overrightarrow{OA} \text{ x } \hat{OB}\right| = \left|\begin{pmatrix}2\\3\\-1\end{pmatrix} \text{ x } \frac{1}{\sqrt{38}}\begin{pmatrix}5\\-2\\3\end{pmatrix}\right| = \frac{1}{\sqrt{38}}\begin{pmatrix}7\\-11\\-19\end{pmatrix} = \sqrt{\frac{531}{38}}$$

$$\overrightarrow{AB} = \begin{pmatrix}3\\-5\\4\end{pmatrix}, \overrightarrow{AP} = \begin{pmatrix}2+3t\\3-5t\\4t-1\end{pmatrix} - \begin{pmatrix}2\\3\\-1\end{pmatrix} = t\begin{pmatrix}3\\-5\\4\end{pmatrix}$$

Since $\overrightarrow{AP} = t\overrightarrow{AB}$, and A is a common point on APB

∴ A,P and B are collinear for all values of t.

$$\overrightarrow{OP}.\overrightarrow{AB} = O$$

$$\begin{pmatrix}2+3t\\3-5t\\4t-1\end{pmatrix}.\begin{pmatrix}3\\-5\\4\end{pmatrix} = 0$$

$$t = \frac{13}{50}$$

2. MJC/1/1

$$\frac{\mathbf{c}\bullet\mathbf{b}}{|\mathbf{c}||\mathbf{b}|}=\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \quad \text{----(1)}$$

Since AC is perpendicular to OB,

$$\overrightarrow{AC}\bullet\overrightarrow{OB}=0$$

$$\left(\overrightarrow{OC}-\overrightarrow{OA}\right)\bullet\overrightarrow{OB}=0$$

$$(\mathbf{c}-\mathbf{a})\bullet\mathbf{b}=0$$

$$\mathbf{c}\bullet\mathbf{b}=\mathbf{a}\bullet\mathbf{b} \quad ---(2)$$

Therefore sub (2) into (1) :

$$\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{c}||\mathbf{b}|}=\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$|\mathbf{c}|=|\mathbf{a}|$$

k is 1.

3. IJC/1/2

(i)	$\overrightarrow{OB}=\dfrac{\overrightarrow{OA}+3\overrightarrow{OM}}{4}$ $\overrightarrow{OM}=\dfrac{4\overrightarrow{OB}-\overrightarrow{OA}}{3}=\dfrac{4\mathbf{b}-\mathbf{a}}{3}$
(ii)	Area of $\Delta OAM=\frac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OM}\right|$ $=\frac{1}{2}\left|\mathbf{a}\times\frac{4\mathbf{b}-\mathbf{a}}{3}\right|$ $=\frac{1}{6}\left|\mathbf{a}\times 4\mathbf{b}-\mathbf{a}\times\mathbf{a}\right| \quad \because(\mathbf{a}\times\mathbf{a})=\mathbf{0}$ $=\frac{2}{3}\left|\mathbf{a}\times\mathbf{b}\right|$

$\overrightarrow{ON} = \frac{2}{5}\mathbf{b}$

$$
\begin{aligned}
&= \frac{1}{2}\left|\overrightarrow{AN} \times \overrightarrow{AB}\right| \\
&= \frac{1}{2}\left|\left(\frac{2}{5}\mathbf{b} - \mathbf{a}\right) \times (\mathbf{b} - \mathbf{a})\right| \\
\text{Area of } \Delta ANB &= \frac{1}{2}\left|-\frac{2}{5}(\mathbf{b} \times \mathbf{a}) - (\mathbf{a} \times \mathbf{b})\right| \quad \because (\mathbf{a} \times \mathbf{a}) \text{ and } (\mathbf{b} \times \mathbf{b}) = \mathbf{0} \\
&= \frac{1}{2}\left|\frac{3}{5}(\mathbf{a} \times \mathbf{b})\right| \\
&= \frac{3}{10}|\mathbf{a} \times \mathbf{b}|
\end{aligned}
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Ratio of ΔANB to ΔOAM $= 9 : 20$