Assignment

1. **Solution:**

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 6 \\ \alpha - 26 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 26 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ 6 \\ \alpha - 26 \end{pmatrix}$$

$1 + 2t = 9 + 4\lambda \quad (1)$

$3 - t = 3 + 6\lambda \quad (2)$

$7 + 5t = 26 + \lambda(\alpha - 26) \quad (3)$

From (1) and (2), $t = 3, \lambda = -\frac{1}{2}$

Subt into (3): $\alpha = 34$

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}.\begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix}}{\sqrt{30}\sqrt{116}} = \frac{42}{\sqrt{30}\sqrt{116}}$$

$\theta = 44.6°$

$$CQ = \left|\left[\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}\right] x \frac{1}{\sqrt{30}}\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}\right| = \left|\left[\begin{pmatrix} 1 \\ 2 \\ -6 \end{pmatrix}\right] x \frac{1}{\sqrt{30}}\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}\right| = \left|\frac{1}{\sqrt{30}}\begin{pmatrix} 4 \\ -17 \\ -5 \end{pmatrix}\right| = \sqrt{11}$$

2. **Solution:**

$$(i)\overrightarrow{AB} = -3\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 10 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$$

$$\frac{x-5}{3} = \frac{z-10}{4}, y = 4$$

$$(ii)\left|\overrightarrow{AC}.\overrightarrow{AB}\right| = \left|\begin{pmatrix}-10\\5\\-5\end{pmatrix}.\frac{1}{5}\begin{pmatrix}3\\0\\4\end{pmatrix}\right| = \left|\frac{-50}{5}\right| = 10$$

$$(iii)\,\text{Re}\,quired\ \text{Distance}=\sqrt{25(6)-100} = 5\sqrt{2}$$

$$\overrightarrow{AN} = 10.\overrightarrow{AB} = 10\frac{-1}{5}\begin{pmatrix}3\\0\\4\end{pmatrix}$$

$$\overrightarrow{ON} - \overrightarrow{OA} = -2\begin{pmatrix}3\\0\\4\end{pmatrix}$$

$$\overrightarrow{ON} = \begin{pmatrix}-1\\4\\2\end{pmatrix}$$

$$(iv)\frac{1}{2}\left[\overrightarrow{OC} + \overrightarrow{OD}\right] = \overrightarrow{ON}$$

$$\overrightarrow{OD} = \begin{pmatrix}3\\-1\\-1\end{pmatrix}$$

3(a)

(i) $\pi : \underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 4$

$\pi_1 : z = 0 \Rightarrow \underset{\sim}{r} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$

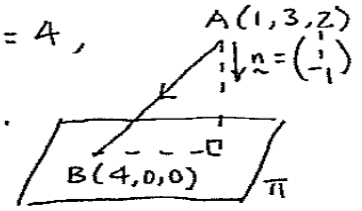
$$\cos\theta = \frac{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{1^2+1^2+1^2}\sqrt{1^2}} = \frac{-1}{\sqrt{3}}$$

$\theta = 125.264°$

$\therefore$ acute $\angle$ betw the two planes is $180° - 125.264° = 54.736° \approx 54.7°$

(ii) Since $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 4$,

B(4,0,0) lies in π.



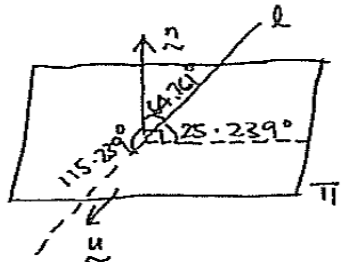
$\perp$ distance

$= |\overrightarrow{AB} \cdot \hat{\underset{\sim}{n}}|$

$= \left| \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right| = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

(b) $\pi : \underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} = 4$

$\ell : \underset{\sim}{r} = \overrightarrow{x}\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$



$$\cos\theta = \frac{\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{1^2+1^2+3^2}\sqrt{1^2+1^2}} = \frac{-2}{\sqrt{11}\sqrt{2}}$$

$\theta = 115.239°$

acute $\angle$ betw ℓ and $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ is $180° - 115.239° = 64.761°$

acute $\angle$ betw ℓ and π is $90° - 64.761° = 25.239° \approx 25.2°$