Binomial Distribution

1

(i) A = no. of ambidextrous people in a sample of 10 ~ B(10, 0.05)

$P\left(A \geq 1\right) = 1 - P\left(A = 0\right) = 1 - 0.59873693 = 0.40126307 = 0.401$

(ii) L = no. of left-handed people in a sample of 10 ~ B(10, 0.15)

$P\left(L < 3\right) = P\left(L \leq 2\right) = 0.82019648 = 0.820$

2

(i) E(X) = np = 22(0.2) = 4.4

(ii) X ~ B(22, 0.2)

P(X ≥ 5) = 1 − P(X ≤ 4)

= 1 − 0.543 = 0.457

(iii) P(X = 3) = 0.17755

P(X = 4) = 0.21084

P(X = 5) = 0.18976

} 4 applicants is most likely

3

(i) Let X be the random variable number of damaged potatoes out of a sample of 12 potatoes.

Then X ~ B(12, 0.1)

$P(X = 3) = {}^{12}C_3\,(0.1)^3\,(0.9)^9$

= 0.0852325 ≈ 0.0852

(ii) Let Y be the random variable number of damaged potatoes out of a sample of *n* potatoes.

Then Y ~ B(*n*, 0.1)

P(Y ≥ 3) > 0.06

1 − P(Y ≤ 2) > 0.06

P(Y ≤ 2) < 0.94

Using GC, binomcdf(9, 0.1, 2) = 0.9470

binomcdf(10, 0.1, 2) = 0.9298

binomcdf(11, 0.1, 2) = 0.9104

Hence, least value of *n* = 10.

4

$p \approx \frac{600}{1500} = \frac{2}{5}$

Let *X* rep the no. of seeds that will germinate out of 24.

$X \sim B(24, \frac{2}{5})$

P(*X* ≥ 12) = 1 − P(*X* ≤ 11)

= 0.213

P(*X* = 2) = 0.209

P(*X* = 3) = <u>0.279</u>

P(*X* = 4) = 0.232

∴Most probable no. that will germinate in a batch of eight is 3.

Let *Y* rep the no. of seeds that will germinate in a batch of 8.

$Y \sim B(8, \frac{2}{5})$

$P(Y > 4) = 1 - P(Y \le 4) = 0.1736704.$

P(only one batch produces more than 4 germinating seeds)

$= 3\,P(Y > 4)\,[P(Y \le 4)]^2$

$= 3(0.1736704)\,[1 - 0.1736704]^2$

$= 0.356$

5 (i) Let X be the number of students from a class that will be late for school in a day

$X \sim B(26,\ 0.08)$

$P(X \ge 3) = 1 - P(X \le 2)$

$= 0.3457357105 = 0.346$ (3 sig. figs)

(ii) Probability = $(0.3457357105) \times (1 - 0.3457357105)^4 = 0.0634$

6

a Let X be the r.v. "number of red balls". $X \sim B(n,\ p)$.

$np = 10.8$ and $npq = 4.32$

$\Rightarrow\ q = \frac{4.32}{10.8} = 0.4,\ \ p = 0.6,\ \ n = \frac{10.8}{0.6} = 18$

$P(X \ge 16)\ =\ 1 - P(X \le 15) =\ 0.00823$

bi $1 - \left(\frac{3}{7}\right)^n > 0.999\ \Rightarrow\ \left(\frac{3}{7}\right)^n < 0.001$

$n > \ln(0.001)/\ln\left(\frac{3}{7}\right)\ \Rightarrow\ n > 8.15$

A sample size of at least 9 clients is needed.

7 **(a)** Let X be the no. of defective televisions

Then $X \sim B(20, 0.1)$

$P(X \le 2) =$ binomcdf (20, 0.1, 2) = 0.677

(b) Required probability$= [1 - P(X \le 2)]^4 = [1 - 0.67693]^4$

$= 0.0109$

(c) $P(X \le 7 \mid X \ge 3) = \dfrac{P(3 \le X \le 7)}{P(X \ge 3)}$

$$= \frac{P(X \le 7) - P(X \le 2)}{1 - P(X \le 2)}$$

$$= \frac{0.999584 - 0.67692}{0.323073} = 0.999$$

8

Let X be the no of defective articles out of 50.

$$X \sim \mathrm{B}(50, 0.02)$$

$\mathrm{E}(X) = 50(0.02) = 1$

$$\begin{aligned} &\mathrm{P}(X \ge 2) \\ &= 1 - \mathrm{P}(X \le 1) \\ &= 0.26423 \\ &= 0.264 (3\text{sf}) \end{aligned}$$

$P(X \le 4 \mid X \ge 2)$

$$= \frac{P(2 \le X \le 4)}{P(X \ge 2)}$$

$$= \frac{0.26102}{0.26423} = 0.988 \text{ (3sf)}$$

9 (i) Let X be the number of employees, out of n, with good performance.

$$X \sim B(n, 0.26)$$

$$\begin{aligned} &P(X \ge 1) \le 0.96 \\ &1 - P(X = 0) \le 0.96 \\ &P(X = 0) \ge 0.04 \\ &\binom{n}{0}(0.26)^0(1 - 0.26)^{n-0} \ge 0.04 \\ &(0.74)^n \ge 0.04 \\ &n \le \frac{\ln(0.04)}{\ln(0.74)} \\ &n \le 10.6902 \\ &\text{greatest } n = 10 \end{aligned}$$

(ii) Let Y be the number of employees, out of 200, with excellent performance.

$$Y \sim B(200, 0.04) \approx N(8, 7.68)$$

since n is large and $np > 5$ and $nq > 5$.

$$P(Y > 7) = P(Y \geq 8) = P(Y > 7.5)$$
$$= 0.572$$

(iii) Let W be the number of employees, out of 20, with average or poor performance.

$$W \sim B(20, 0.7)$$

$$P(W \geq 12 | W < 17)$$
$$= \frac{P(W \geq 12 \cap W < 17)}{P(W < 17)}$$
$$= \frac{P(12 \leq W \leq 16)}{P(W \leq 16)}$$
$$= \frac{P(W \leq 16) - P(W \leq 11)}{P(W \leq 16)}$$
$$= \frac{0.8929131955 - 0.1133314628}{0.8929131955}$$
$$= 0.873$$

10i	Let X = "no of interviewers that will accept the candidate in the first round" X ~ B (10, 0.9) $P(X \geq 9) = 1 - P(X \leq 8)$ $= 1 - 0.26390107$ $= 0.736$
9ii	Prob of qualifying for a scholarship = $0.73609893 \times (0.8)^5 = 0.241$ (3.*s.f.*)
	Let W = "no of candidates that qualify for the scholarship out of *n* candidates" W ~ B(n ,0.241) $P(W \geq 2) \geq 0.98$
iii	$1 - P(W = 0) - P(W = 1) \geq 0.98$ $1 - (0.759)^n - n(0.241)(0.759)^{n-1} \geq 0.98$ Using GC, $n \geq 21.7$ Least n = 22