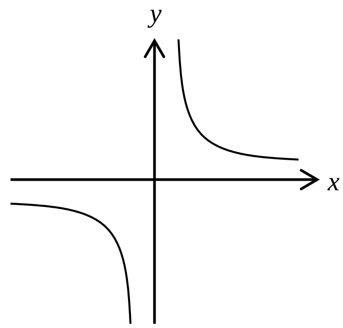
Graphing Techniques (I)

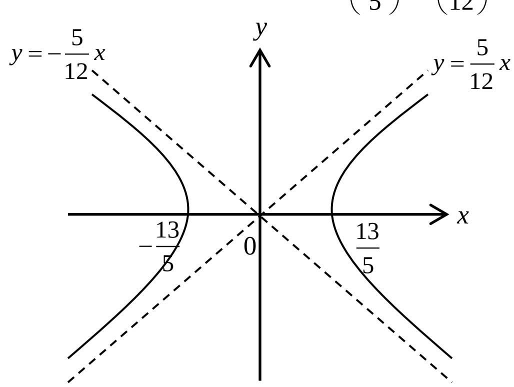
1. (a) $xy = 2 \Rightarrow y = \dfrac{2}{x}$



Shape: Rectangular Hyperbola

Hori asym: $y = 0$ Vert asym: $x = 0$

(e) $(5x)^2 - (12y)^2 = 13^2 \Rightarrow \dfrac{x^2}{\left(\frac{13}{5}\right)^2} - \dfrac{y^2}{\left(\frac{13}{12}\right)^2} = 1$



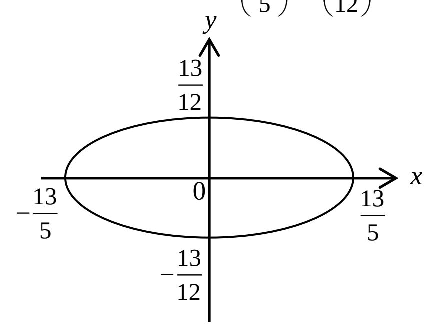
Shape: Hyperbola

Oblique asym: $y = \pm\dfrac{5}{12}$

Intersections with x-axis: $\left(-\dfrac{13}{5}, 0\right)$, $\left(\dfrac{13}{5}, 0\right)$

(c) $y = \dfrac{3x^2 + 2x + 4}{x - 4} = 3x + 14 + \dfrac{60}{x - 4}$

(d) $(5x)^2 + (12y)^2 = 13^2 \Rightarrow \dfrac{x^2}{\left(\frac{13}{5}\right)^2} + \dfrac{y^2}{\left(\frac{13}{12}\right)^2} = 1$

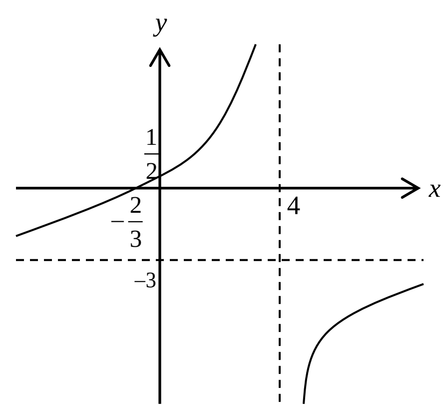


Shape: Ellipse centre (0,0)

x-intercepts: $\left(-\dfrac{13}{5}, 0\right), \left(\dfrac{13}{5}, 0\right)$

y-intercepts: $\left(0, -\dfrac{13}{12}\right), \left(0, \dfrac{13}{12}\right)$

(b) $y = \dfrac{3x + 2}{4 - x} = -3 + \dfrac{14}{4 - x}$



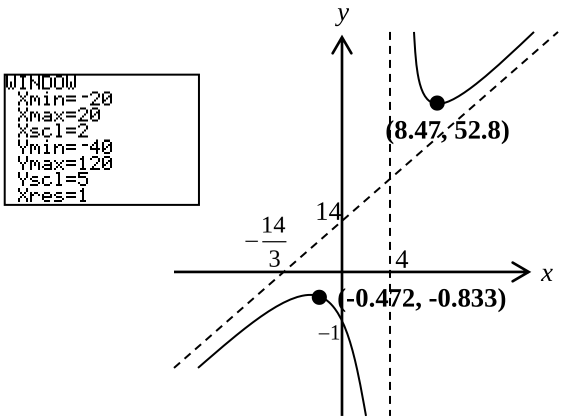
Shape: Rectangular Hyperbola

Hori asym: $y = -3$ Vert asym: $x = 4$

Intersections with x-axis: $\left(-\dfrac{2}{3}, 0\right)$

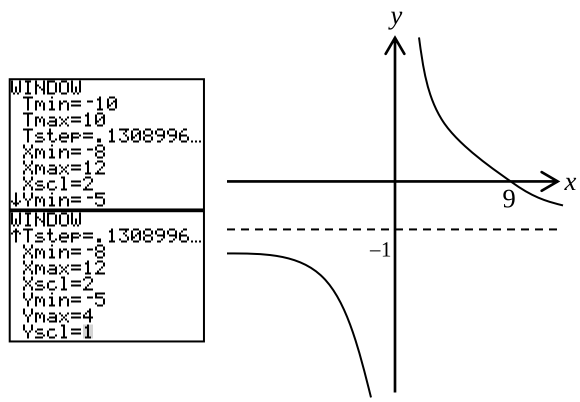
Intersections with y-axis: $\left(0, \dfrac{1}{2}\right)$

(f) $x = 3t$, $y = \dfrac{3}{t} - 1 \Rightarrow y = \dfrac{9}{x} - 1$



Oblique asym: $y = 3x + 14$
Vert asym: $x = 4$

Intersections with y-axis: (0,-1)
Min pt. : (8.47, 52.8)
Max. pt. : (-0.472, -0.833)



Shape: Rectangular Hyperbola
Intersections with x-axis: (9,0)
Hori asym: $y = -1$

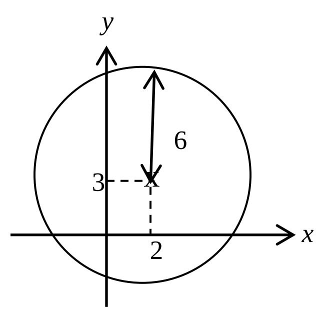
(g) $2x^2 + 2y^2 - 8x - 12y - 46 = 0$

$\Rightarrow \ldots \Rightarrow (x-2)^2 + (y-3)^2 = 6^2$ -(*)

Consider C: $x^2 + y^2 = 6^2$.

(*) is a translation of C 2 units to the right

left
and 3 units upwards.



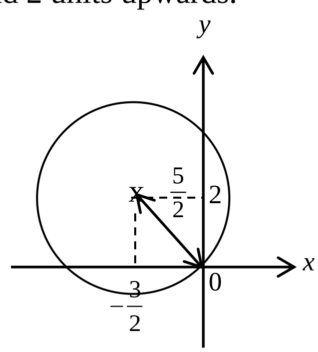
(h) $x(x+3) + y(y-4) = 0$

$\Rightarrow \ldots \Rightarrow \left(x + \frac{3}{2}\right)^2 + (y-2)^2 = \left(\frac{5}{2}\right)^2$ -(*)

Consider C: $x^2 + y^2 = \left(\frac{5}{2}\right)^2$.

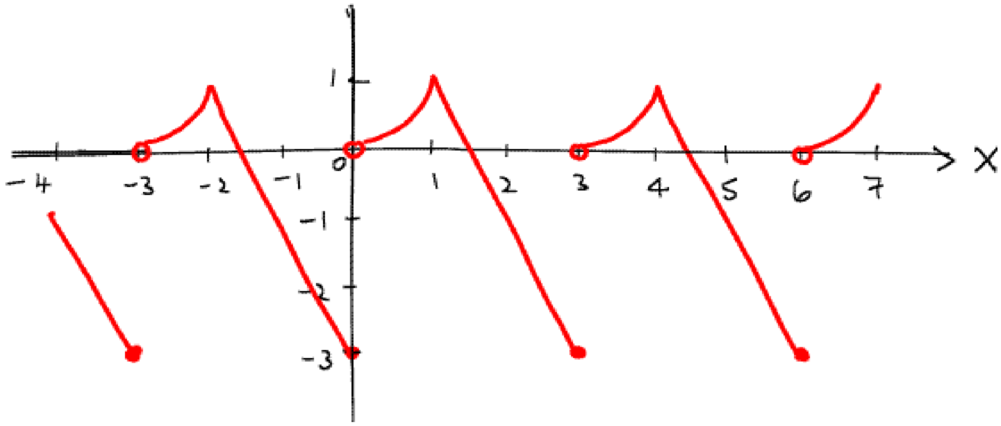
(*) is a translation of C $\frac{3}{2}$ units to the

And 2 units upwards.



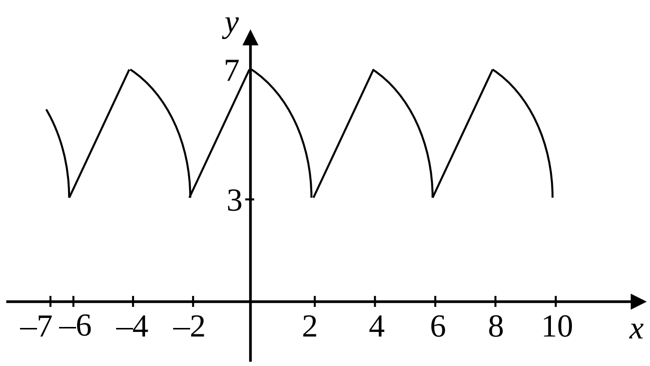
Check that (*) passes through the origin!

3.



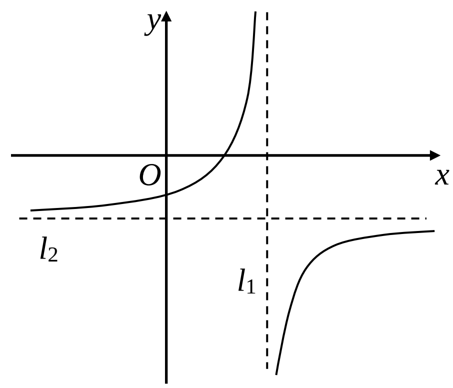
f(26) = f(24 + 2) = f(2) = –1

4.

4i) f(27) + f(45) = f(23) + f(41) = f(19) + f(37) : : = f(3) + f(1) = 5 + 6 = 11	
(ii)  	**Teaching Point:** Students should be advised to sketch a clear and properly–labelled graph.

5.

(i) $l_1 : x = 3$; $l_2 : y = -2$



(ii)

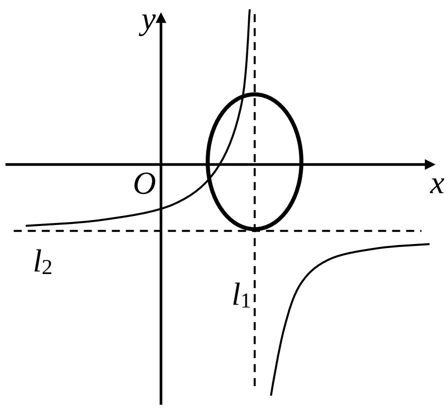
$$4(x-3)^4 + (5-2x)^2 - 4(x-3)^2 = 0$$

$$4(x-3)^2 + \left(\frac{5-2x}{x-3}\right)^2 - 4 = 0$$

$$(x-3)^2 + \frac{1}{4}\left(\frac{5-2x}{x-3}\right)^2 = 1$$

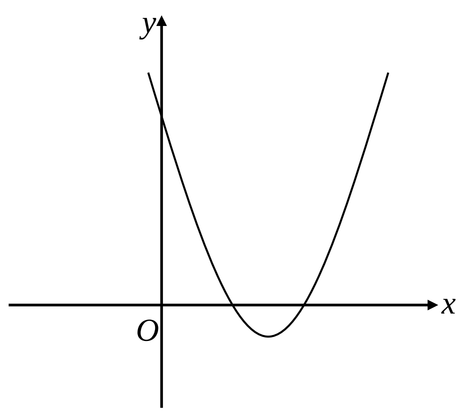
$$(x-3)^2 + \frac{1}{4}y^2 = 1$$

Hence the additional graph is $(x-3)^2 + \frac{1}{4}y^2 = 1$. From the graph, there are 2 points of intersection. Therefore, there are 2 real roots.

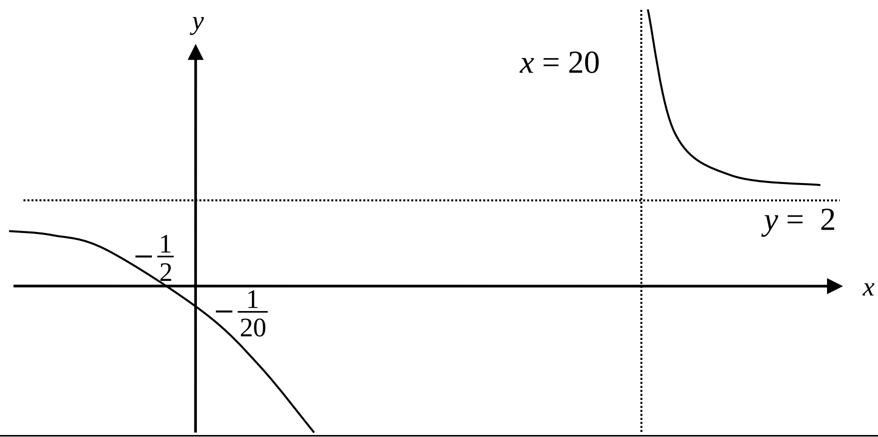


Alternative method:

Sketch the graph of $y = 4(x-3)^4 + (5-2x)^2 - 4(x-3)^2$

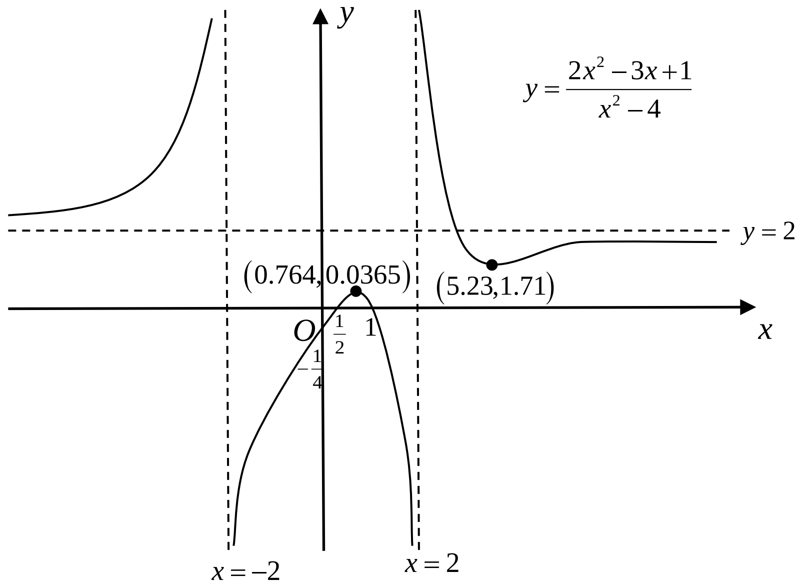


6.

9i)	$B = 20$ $y = 2 + \dfrac{41}{20 - x} \Rightarrow \quad A = -2$
(ii)	

7	Graphing Techniques
(i)	Let $y = \dfrac{2x^2 - 3x + 1}{x^2 - 4}$ $yx^2 - 4y = 2x^2 - 3x + 1$ $(2 - y)x^2 - 3x + (4y + 1) = 0$ For real roots of *x*, Discriminant ≥ 0 $(-3)^2 - 4(2 - y)(4y + 1) \geq 0$ $9 - 4(8y + 2 - 4y^2 - y) \geq 0$ $16y^2 - 28y + 1 \geq 0$ For $16y^2 - 28y + 1 = 0$ $y = \dfrac{28 \pm \sqrt{(-28)^2 - 4(16)}}{2(16)}$ $= \dfrac{28 \pm \sqrt{720}}{32}$ $= \dfrac{7 \pm 3\sqrt{5}}{8}$ $\therefore 16y^2 - 28y + 1 \geq 0$ $y \leq \dfrac{7 - 3\sqrt{5}}{8}$ or $y \geq \dfrac{7 + 3\sqrt{5}}{8}$ Hence, C cannot lie between $\dfrac{7 - 3\sqrt{5}}{8}$ and $\dfrac{7 + 3\sqrt{5}}{8}$

(ii)

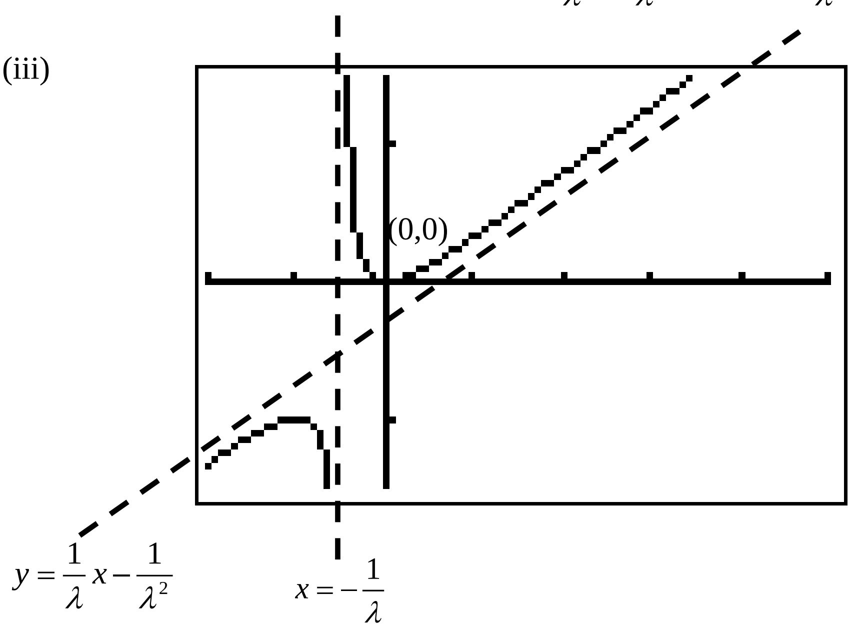


9. (i)(0,0)

(ii) $y=\frac{x^2}{\lambda x+1}=\frac{1}{\lambda}x-\frac{1}{\lambda^2}+\frac{1}{\lambda^2(\lambda x+1)}$.

The equations of asymptotes are: $y=\frac{1}{\lambda}x-\frac{1}{\lambda^2}$ and $x=-\frac{1}{\lambda}$.

(iii)



10.

7i) $c = -3$

ii) $a = 1$

$$y = \frac{x^2 + bx + 16}{x - 3} = x + 11 + \frac{A}{x - 3}$$

$\Rightarrow \quad x^2 + bx + 16 = (x + 11)(x - 3) + A$

Comparing coefficients of x,

$b = -3 + 11 = 8$

iii) Using GC, minimum point = (10.0, 28.0)
Maximum point = (-4.0, 0)

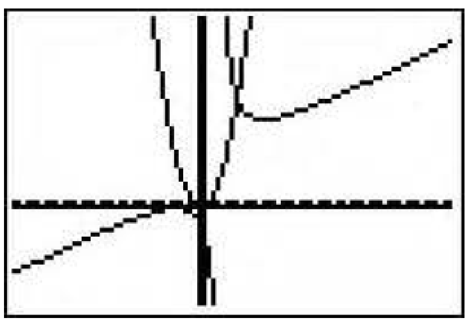
iv) $x^3 - 4x^2 - 8x - 16 = 0$

$\Rightarrow \quad x^3 - 3x^2 = x^2 + 8x + 16$

$\Rightarrow \quad x^2(x - 3) = x^2 + 8x + 16$

$\Rightarrow \quad x^2 = \dfrac{x^2 + 8x + 16}{x - 3}$

$\therefore k = 1$



The graphs of $y = x^2$ and $y = \dfrac{x^2 + 8x + 16}{x - 3}$ intersect at one point only. So the equation $x^3 - 4x^2 - 8x - 16 = 0$ has exactly one real root.