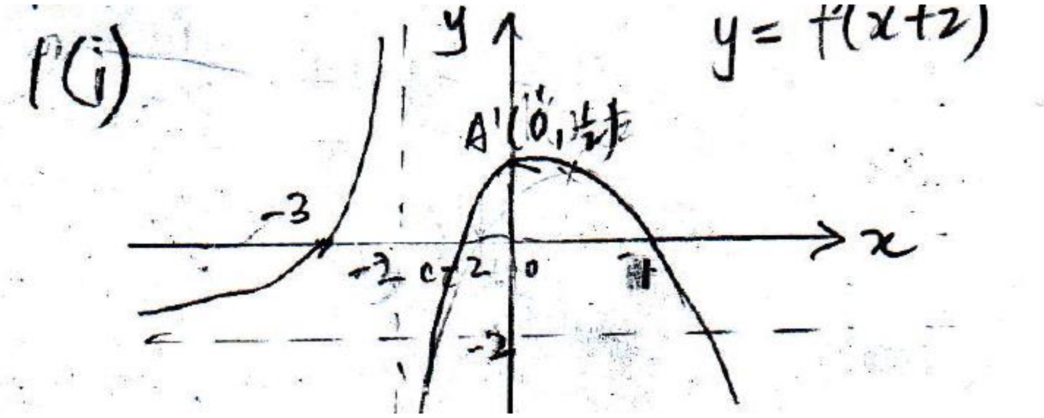
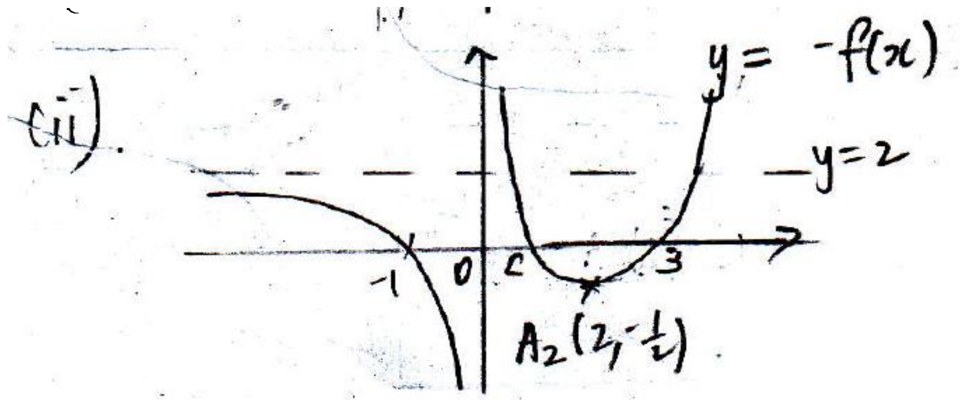
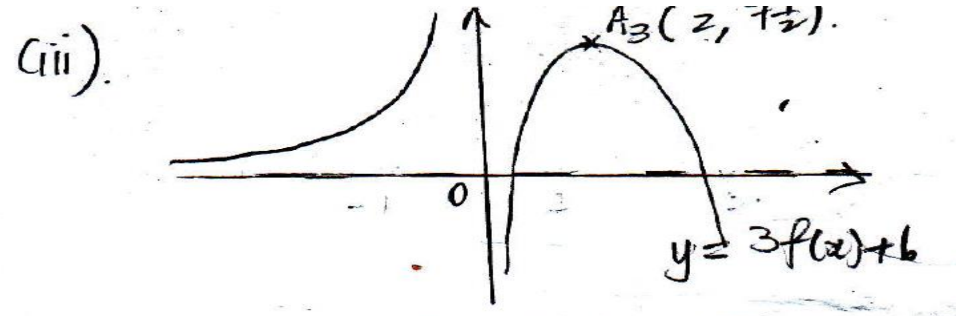
Graphing Techniques (II)



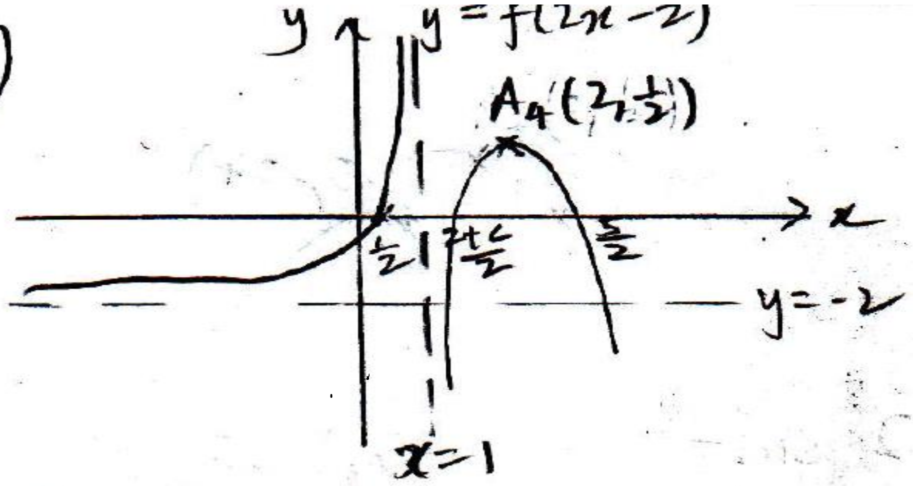




$A(2, \frac{1}{2}) \rightarrow A_3(2, \frac{3}{2}+6)$

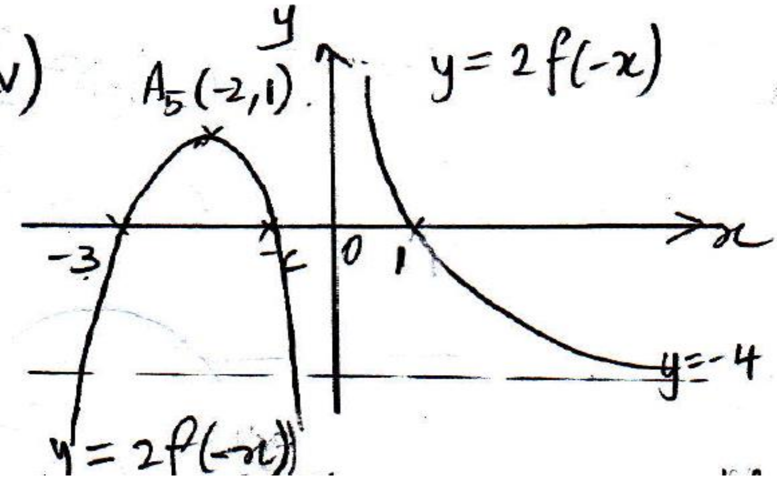
$y = -2 \rightarrow y = -2 \times 3 + 6 = 0$

(iv)



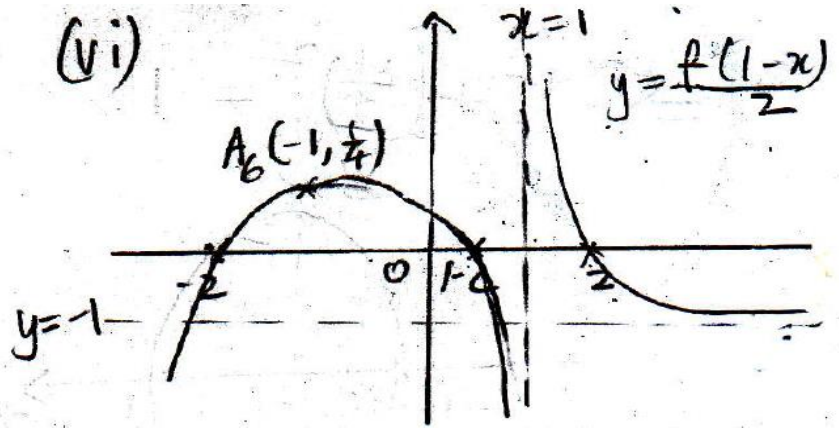
y
y = f(2x − 2)
A4(2, 1/2)
x
1/2
1
y = −2
x = 1

(v)



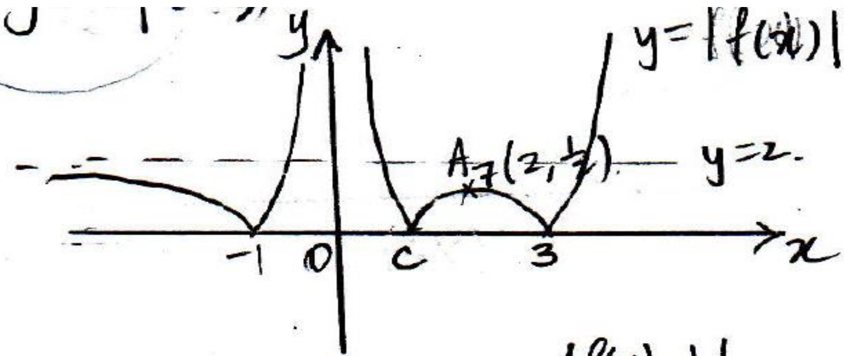
y
A5(−2,1)
y = 2f(−x)
x
−3
0
1
y = −4
y = 2f(−x)

(vi)

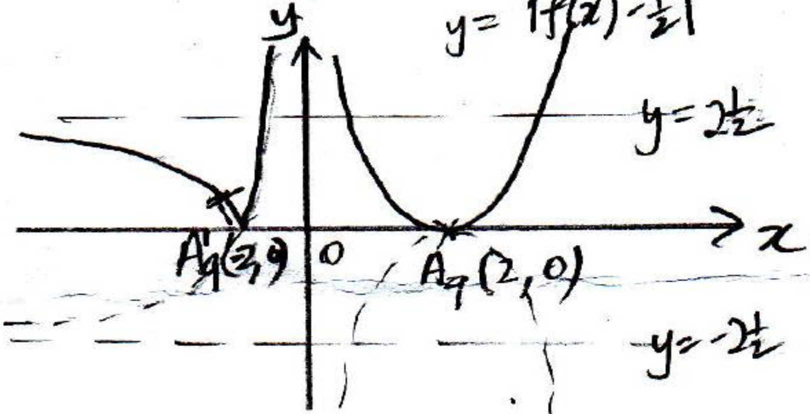


x = 1
y = f(1−x)/2
A6(−1, 1/4)
−2
0
2
y = −1

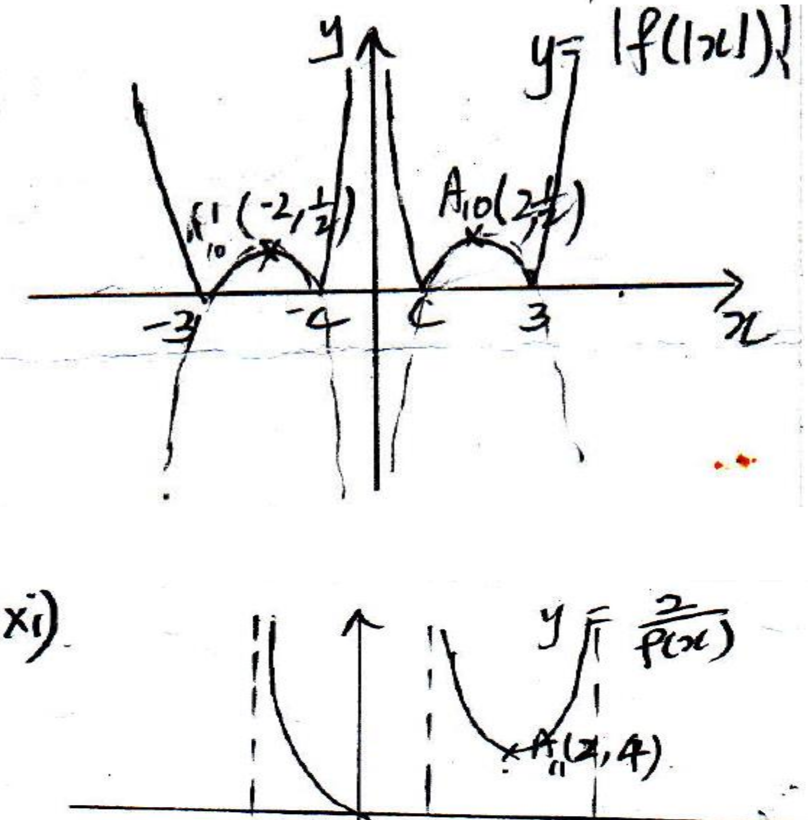
(vii)


y
$y = |f(x)|$
$A_7(2, \frac{1}{2})$
$y = 2$
-1
0
c
3
x

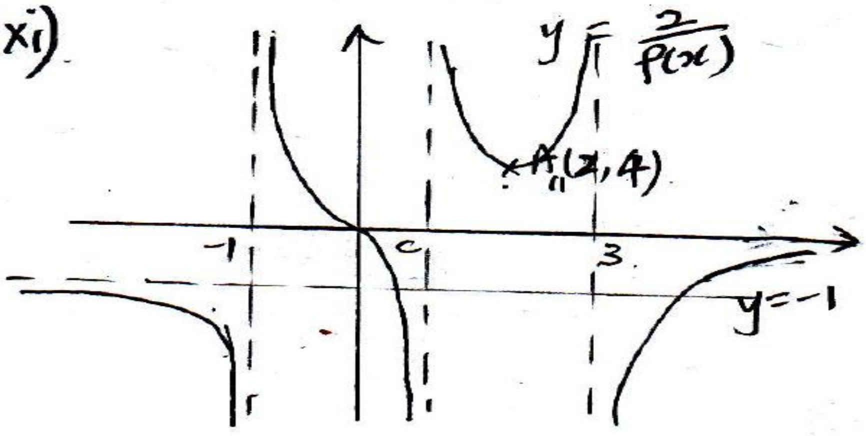
(ix)


y
$y = |f(x) - \frac{1}{2}|$
$y = 2\frac{1}{2}$
x
0
$A_7(2, 0)$
$y = -2\frac{1}{2}$

(x)


y
$y = |f(|x|)|$
$A'_{10}(-2, \frac{1}{2})$
$A_{10}(2, \frac{1}{2})$
-3
$-c$
c
3
x

(xi)


$y = \frac{2}{f(x)}$
$A_{11}(2, 4)$
-1
c
3
$y = -1$

2.

(i) $c = 2$, $a = 2$

$$y = \frac{ax+b}{x+2} = 2 + \frac{k}{x+2} = 2 + \frac{4}{x+2}$$

$ax + b = 2(x+2) + k$

$a = 2$

$b = 4 + k \Rightarrow k = -4$

At $(0,0)$, $b = 0$.

(ii) $\underset{=f(x)}{y = \frac{1}{x}} \xrightarrow{A} \underset{=f(x+2)}{y = \frac{1}{x+2}} \xrightarrow{B} y = \frac{-4}{x+2} \xrightarrow{C} \boxed{y = 2 + \frac{4}{x+2}}$

A : Translation along x-axis by -2 units

B : Scaling " y-axis by factor 4 & Reflection " x- "

C : Translation y-axis " 2 units.

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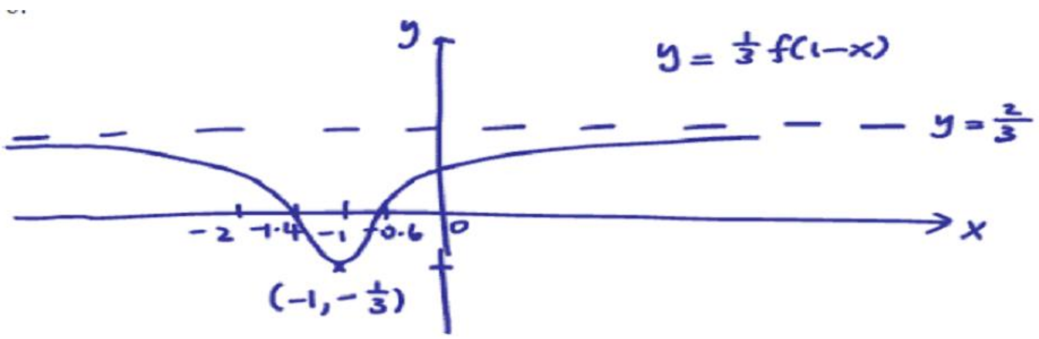
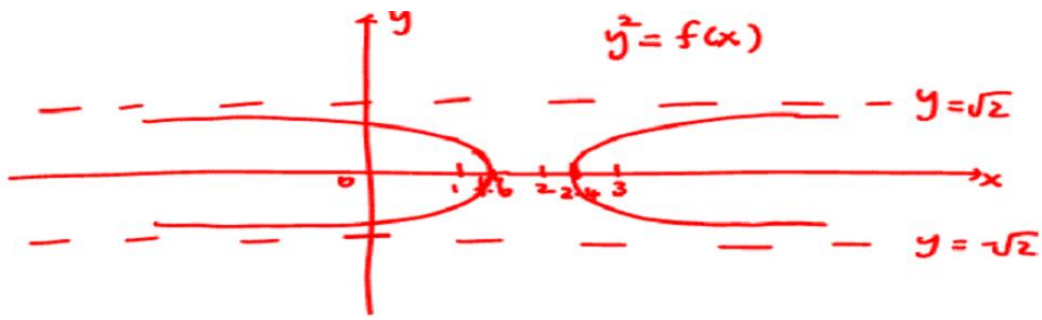
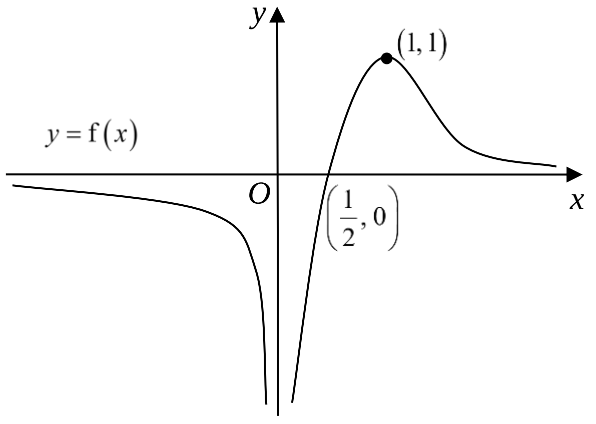
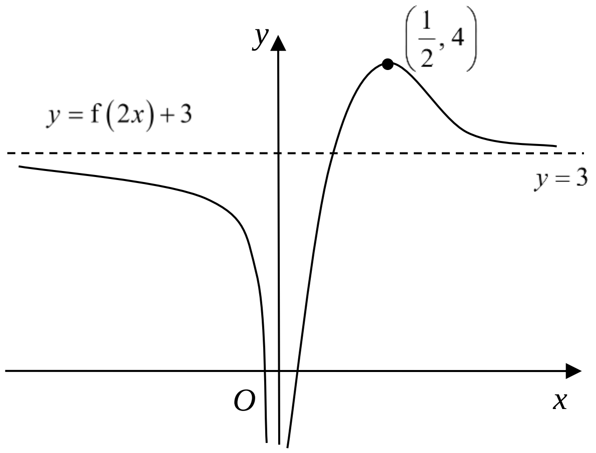
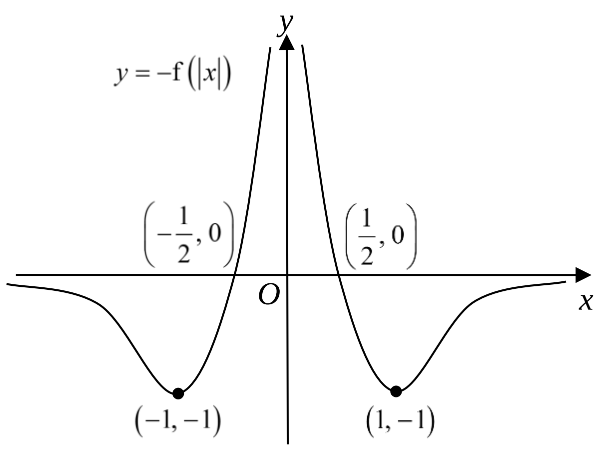
3. (i)



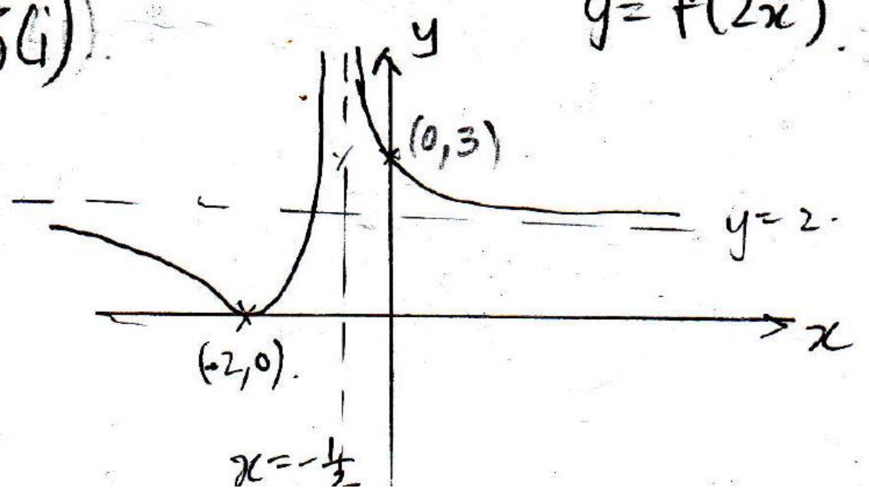
(ii)



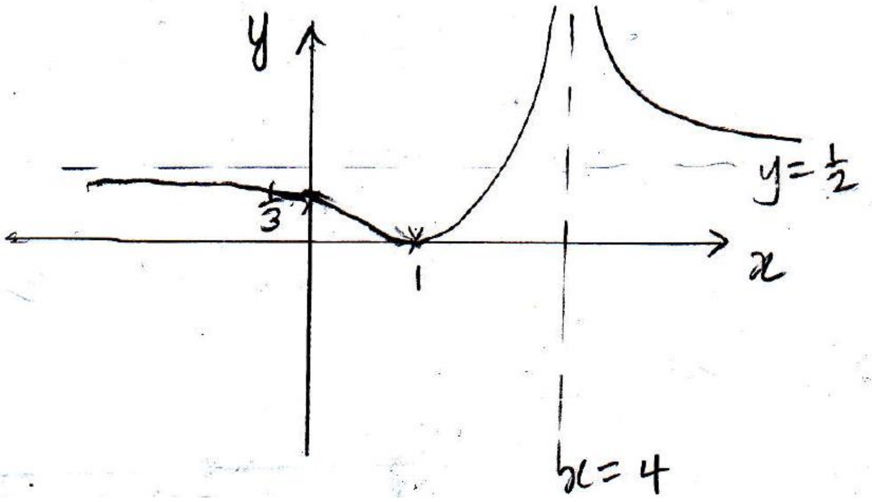
4.

(i)	y $(1, 1)$ $y = \mathrm{f}(x)$ O $\left(\frac{1}{2}, 0\right)$ x
(ii)	$\left(\frac{1}{2}, 4\right)$ y $y = \mathrm{f}(2x) + 3$ $y = 3$ O x
(iii)	y $y = -\mathrm{f}(\lvert x \rvert)$ $\left(-\frac{1}{2}, 0\right)$ $\left(\frac{1}{2}, 0\right)$ O x $(-1, -1)$ $(1, -1)$

5(i)

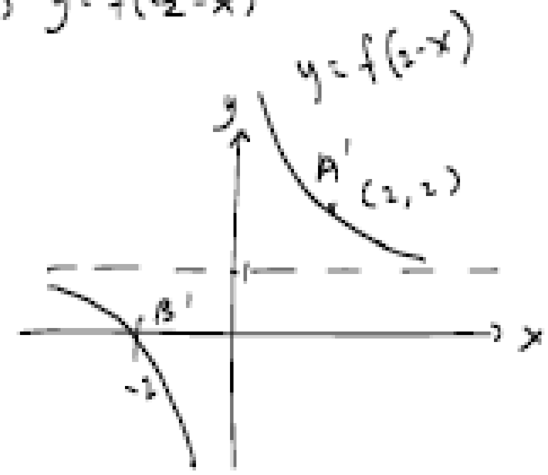


(ii)

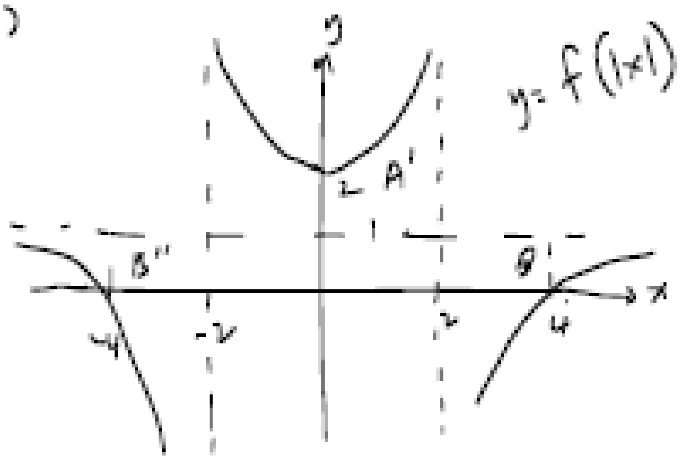


6(a).

(i) (i) $y = f(-2-x)$



(ii)



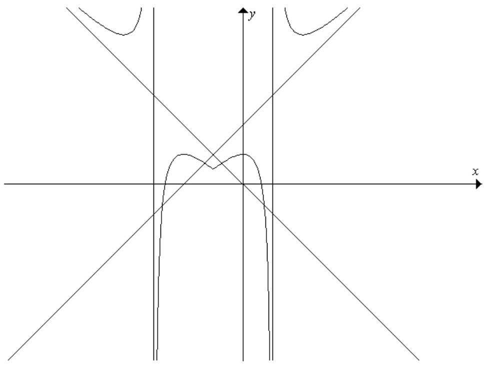
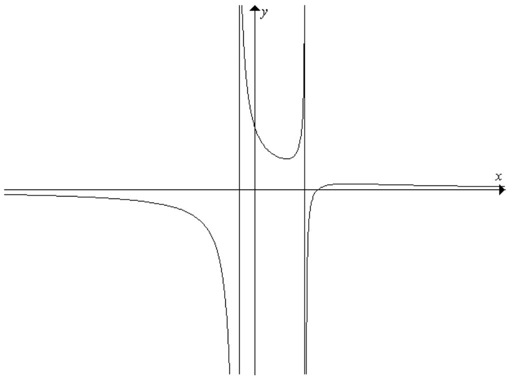
(b) $\frac{1}{2}y = e^{-2x+6} \Rightarrow y = 2e^{-2x+6} = g(x)$

Before C : $(y \times \frac{1}{2})$: $y = e^{-2x+6}$

Before B : $(x \to x+3)$: $y = e^{-2(x+3)+6} = e^{-2x}$

Before A : $(x \to -x)$: $y = e^{2x}$

$\therefore \quad y = f(x) = e^{2x}$

7(i) $y = \mathrm{f}\left(\lvert x+1 \rvert\right)$ 	**B1** Correct shape of graph **B1** Correct intercepts (0.5,0) and (-2.5,0) **B1** Correct turning points: State either (0,1) or (-2,1), and either (2,5) or (-4,5) **B1** Correct vertical asymptotes x = 1 and x = -3 **B1** Correct horizontal asymptotes: State either $y = x+2$ or $y = -x$
(ii) $y = \dfrac{1}{\mathrm{f}(x)}$ 	**B1** Correct shape of graph **B1** Correct intercept (2,0) **B1** Correct tuning points (1,1) and (3, 0.2) **B1** Correct vertical asymptotes x = -0.5 and x = 1.5 **B1** Correct horizontal asymptote y = 0

8.

(a)	$x^2+y^2=1$ $\downarrow$ Replace x by $\frac{x}{2}$ $\left(\frac{x}{2}\right)^2+y^2=1$ $\downarrow$ Replace x by $x-1$ $\left(\frac{x-1}{2}\right)^2+y^2=1$ $\downarrow$ Replace y by $\frac{y}{2}$ $\left(\frac{x-1}{2}\right)^2+\left(\frac{y}{2}\right)^2=1$ $\Rightarrow (x-1)^2+y^2=4$

The transformations are scaling parallel to x-axis with a scale factor 2, followed by a translation of 1 units in the direction of x-axis and scaling parallel to y-axis with a scale factor 2.

Alternative Solution

$$x^2+y^2=1$$

$$\Big\downarrow \quad \text{Replace } x \text{ by } x-\frac{1}{2}$$

$$\left(x-\frac{1}{2}\right)^2+y^2=1$$

$$\Big\downarrow \quad \text{Replace } x \text{ by } \frac{x}{2}$$

$$\left(\frac{x}{2}-\frac{1}{2}\right)^2+y^2=1$$

$$\Big\downarrow \quad \text{Replace } y \text{ by } \frac{y}{2}$$

$$\left(\frac{x-1}{2}\right)^2+\left(\frac{y}{2}\right)^2=1$$

$$\Rightarrow (x-1)^2+y^2=4$$

The transformations are translation of $\frac{1}{2}$ units in the direction of x-axis, followed by scaling parallel to x-axis with a scale factor 2 and scaling parallel to y-axis with a scale factor 2.

(b)

$$y=\frac{2x^2+ax-7}{x-1}=2x+a+2+\frac{a-5}{x-1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{a-5}{(x-1)^2}$$

At $\frac{\mathrm{d}y}{\mathrm{d}x}=0$,

$$2-\frac{a-5}{(x-1)^2}=0$$

$$2(x-1)^2-a+5=0$$

$$2x^2-4x+7-a=0$$

For $\mathrm{f}(x)$ to have no turning point,

$$(-4)^2-4(2)(7-a)<0$$

$$16-56+8a<0$$

$$a<5$$

<table>
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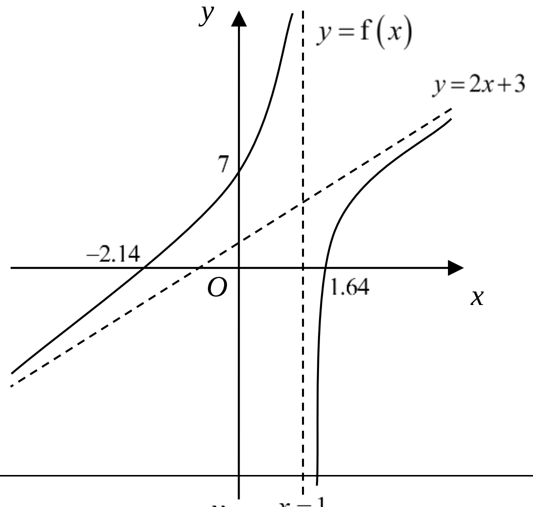
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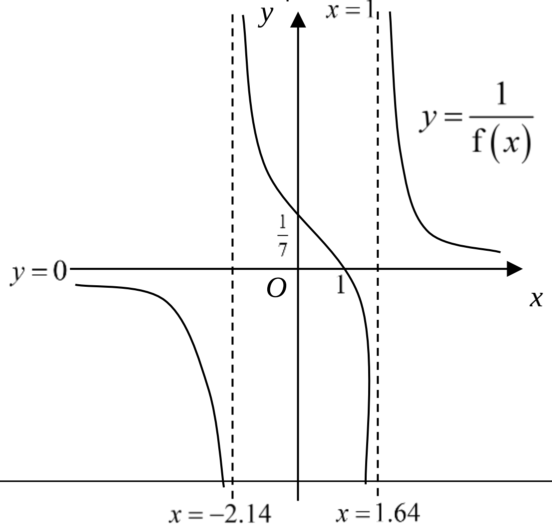
<tr><td>(ii)</td><td>
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