Normal Distribution

1	Let M minutes be the time taken to milk a Magnolia cow, and D minutes be the time taken to milk a Daisy cow. $M \sim \mathrm{N}\left(30, 2^2\right),\ D \sim \mathrm{N}\left(5.5, 0.5^2\right)$
(i)	$\mathrm{P}\left(M < a\right) = 0.85$ From GC, a = 32.073 = 32.1 (3 s.f.)
(ii)	$\mathrm{E}\left(M_1 + M_2 - 11D\right) = 2\times 30 - 11\times 5.5 = -0.5$ $\mathrm{Var}\left(M_1 + M_2 - 11D\right) = 2\times 2^2 + 11^2 \times 0.5^2 = 38.25$ $M_1 + M_2 - 11D \sim \mathrm{N}\left(-0.5, 38.25\right)$ Reqd prob = $\mathrm{P}\left(M_1 + M_2 - 11D \geq 3\right)$ = 0.28573 = 0.286 (3 s.f.)

2

Let D be the mass of a durian

$D \sim N(1.6, 0.2^2)$

Let M be the mass of a durian

$M \sim N(0.3, 0.05^2)$

(i)

Let $P(D > m) = 0.8$

Using GC, $m = 1.43167 \approx 1.43$ kg.

(ii)

$D_{(3)} + M_{(4)} \sim N(3\times 1.6 + 4\times 0.3, 3\times 0.2^2 + 4\times 0.05^2) = N(6, 0.13)$

$P(D_{(3)} + M_{(4)} > 6.5) = 0.08275892 \approx 0.0828$

(iii) $\overline{D} \sim N\left(1.6, \dfrac{0.2^2}{n}\right)$

$P(\overline{D} < 1.45) = 0.0122$

$$P\left(Z < \frac{1.45 - 1.6}{\frac{0.2}{\sqrt{n}}} \right) = 0.0122$$

$$\frac{1.45 - 1.6}{\frac{0.2}{\sqrt{n}}} = -2.2507717$$

$n \approx 9$

(iv)

$8D_{(3)} + 3M_{(4)} \sim N(8 \times 4.8 + 3 \times 1.2, 8^2 \times 3 \times 0.2^2 + 3^2 \times 4 \times 0.05^2)$
$= N(42, 7.77)$
$P\left(8D_{(3)} + 3M_{(4)} < 45\right) \approx 0.859$

3

(i)	Let M be the mass of a randomly chosen Munchi pear. $M \sim \mathrm{N}\left(120, 10^2\right)$ Using GC, $\mathrm{P}\left(100 < M < 126\right) \approx 0.7030$ $\mathrm{P}\left(M < 115\right) \approx 0.3085$ Required probability $= \left(0.7030\right)\left(0.3085\right) \times 2! \approx 0.434$ (3 s.f.)
(ii)	Using GC, $\mathrm{P}\left(M > 122\right) \approx 0.4207$ Let S be the number of pears of mass that is more than 126g. $S \sim \mathrm{B}\left(10, 0.4207\right)$ $\mathrm{P}\left(S = 3\right) \approx 0.196$ (3 s.f.)
(iii)	Let F be the mass of a randomly chosen Fuchi apple. $F \sim \mathrm{N}\left(115, 8^2\right)$ $F_1 + F_2 + F_3 - 3M \sim \mathrm{N}\left(\left(3 \times 115 - 3 \times 120\right), \left(3 \times 8^2 + 9 \times 10^2\right)\right)$ $\Rightarrow F_1 + F_2 + F_3 - 3M \sim \mathrm{N}\left(-15, 1092\right)$ Using GC, Required probability $= \mathrm{P}\left(F_1 + F_2 + F_3 > 3M\right)$ $= \mathrm{P}\left(F_1 + F_2 + F_3 - 3M > 0\right) \approx 0.325$ (3 s.f.)

(iv)	$F \sim \mathrm{N}\left(115, 8^2\right)$ $\Rightarrow \bar{F} \sim \mathrm{N}\left(115, \frac{8^2}{n}\right)$ $\mathrm{P}\left(\left\|\bar{F} - 115\right\| > 4\right) \le 0.3$ $\Rightarrow \mathrm{P}(\bar{F} - 115 < -4) + \mathrm{P}(\bar{F} - 115 > 4) \le 0.3$ $\mathrm{P}\left(Z < -\frac{4}{\sqrt{8^2/n}}\right) + \mathrm{P}\left(Z > \frac{4}{\sqrt{8^2/n}}\right) \le 0.3$ $\therefore \mathrm{P}\left(-\frac{\sqrt{n}}{2} < Z < \frac{\sqrt{n}}{2}\right) \ge 0.7$ $\Rightarrow \mathrm{P}\left(Z < \frac{\sqrt{n}}{2}\right) \ge 0.85$ $\mathrm{invNorm}(0.85) = 1.036$ $\frac{\sqrt{n}}{2} \ge 1.036 \Rightarrow n \ge 4.296$ Hence, least $n = 5$.

4

(i) Given that P(Y < 80) = P(Y > 150)

$$\Rightarrow 120 - k = \frac{80+150}{2} = 115$$
$$\therefore \quad k = 5$$

(ii) $\mathrm{P}(Y < q_L).\mathrm{P}(Y > m).[\mathrm{P}(q_L \le Y \le m)]^3.\frac{5!}{3!}$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^3 \frac{5!}{3!}$$
$$= 0.0390625$$

(iii) $Y_1 + Y_2 - \frac{3}{2}F \sim \mathrm{N}(-40,\ 1100)$

$$\mathrm{P}\left(\left|Y_1 + Y_2 - \frac{3}{2}F\right| \le 5\right)$$
$$= \mathrm{P}(|Z| \le \frac{45}{\sqrt{1100}})$$
$$= \mathrm{P}(-1.356801051 \le Z \le 1.356801051) = 0.825 \text{ (3 s.f.)}$$

Assumption: The weight of a Yummy cereal bar is independent of the weight of a Fullness cereal bar.

(iv) $0.02F \sim N(3.6,\ 0.16)$
$P(0.02F > 3.5) \approx 0.59871 = 0.599$ (3 s.f)

5

(i) Let M denote the r.v of the mass of a male hornbill.
Let F denote the r.v of the mass of a female hornbill.

$M \sim N\left(3500, 150^2\right)$ $\quad\quad F \sim N\left(3000, \sigma^2\right)$

$P\left(F > 3200\right) = 0.05$

$P\left(F \le 3200\right) = 0.95$

$P\left(Z \le \dfrac{3200-3000}{\sigma}\right) = 0.95$

$\dfrac{200}{\sigma} = 1.64485 \Rightarrow \sigma = 121.59 = 122$ (3s.f.)

(ii) $M_1 - M_2 \sim N(0, 2\times 150^2)$

$P\left(\left|M_1 - M_2\right| \ge 100\right) = P\left(M_1 - M_2 \ge 100\right) + P\left(M_1 - M_2 \le -100\right)$

$= 1 - 2P\left(M_1 - M_2 \le -100\right)$

=0.36265=0.363 (3 s.f.)

(iii) Let $T = F_1 + F_2 + F_3 + F_4 + F_5 - 2\left(M_1 + M_2\right)$

$T \sim N\left(5\times 3000 - 4\times 3500, \quad 5\times 121.49^2 + 2^3\times 150^2\right)$

$P\left(T > 0\right) \approx 0.976$

(iv) Probability required

$= \dfrac{4!}{2!2!}\left[P(M > 3600)\right]^3\left[P(M \le 3600)\right]^2$

=0.053967

=0.0540 (3 s.f.)

6

Let A be the timing of swimmer A for the 100m freestyle. $A \sim N(48.0, 0.5^2)$

Let B be the timing of swimmer B for the 100m freestyle. $B \sim N(47.2, 0.8^2)$

(i) **<u>Mtd1</u>**

Required Probability = 1– P(non among the two broke the world record)

$$= 1 - P(A \geq 46.9)P(B \geq 46.9)$$

$$= 1 - (0.986097)(0.646170) = 0.36281 \approx 0.363$$

<u>Mtd2</u>

Required Probability = P(A breaks record) + P(B breaks record) – P(both break record)

$$= P(A < 46.9) + P(B < 46.9) - P(A < 46.9)P(B < 46.9)$$

$$\approx 0.363$$

<u>Mtd3</u>

Required Probability = P(A breaks record, B don't break record)
+ P(A don't break record, B breaks record) + P(both break record)

$$= P(A < 46.9, B \geq 46.9) + P(A \geq 46.9, B < 46.9)$$
$$+ P(A < 46.9)P(B < 46.9)$$

$$\approx 0.363$$

(ii) $A - B \sim N(0.8, 0.89)$

$P(A > B) = P(A - B > 0) = 0.801781 \approx 0.802$

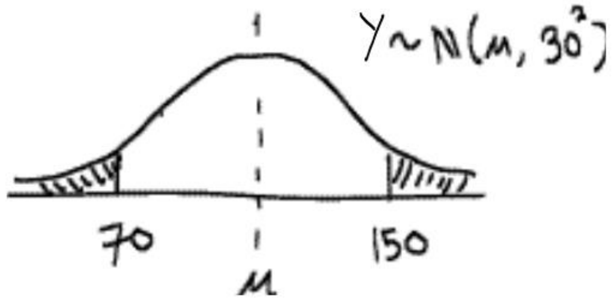
(iii) $20 \times (1 - 0.801781) \approx 4$

Expected times out of 20 is 4 times.

(iv) Let $W = A_1 + A_2 + A_3 + A_4 - 4B \sim N(3.2, 11.24)$

$P(A_1 + A_2 + A_3 + A_4 > 4B) = P(W > 0) = 0.83008 \approx 0.830$

7 First part	$X \sim \text{N}(190, 576)$ $T = 0.001(X_1 + ... + X_{20}) - 0.001(2)(X_{21} + ... + X_{30}) \sim \text{N}(0, 0.03456)$ $\text{P}(\lvert T \rvert \leq 0.15) = \text{P}(-0.15 \leq T \leq 0.15)$ = 0.580 OR $A = X_1 + ... + X_{20} - 2(X_{21} + ... + X_{30}) \sim \text{N}(0, 34560)$ $\text{P}(\lvert A \rvert \leq \frac{0.15}{0.001}) = \text{P}(-150 \leq A \leq 150)$ = 0.580

(i)	Let Y be the r.v. denoting the mass of a randomly chosen apple from Mark's orchard. $Y \sim \mathrm{N}(\mu, 30^2)$  Since the shaded area is the same, using the symmetric property of the normal curve, $\mu = 110$
(ii)	Probability that Mark will get an apple graded as 'large' chosen at random = $P(Y > 150)$ = 0.09121128 Let A be the r.v. denoting the number of apples graded as large out of 65 randomly chosen apples. $A \sim B(65,\ 0.09121128)$ $P(A \geq 5) = 1 - P(A \leq 4)$ $= 0.718$

8

Let r.v. A be the mass of a snapper fish and r.v. B be the mass of a pomfret fish.

$A \sim N(1, 0.1^2)$; $B \sim N(0.6, 0.05^2)$

(a)(i) $A_1 + A_2 + A_3 + B_1 + B_2 \sim N(4.2, 0.035)$

$P[A_1 + A_2 + A_3 + B_1 + B_2 > 4.5] = 0.0544$

(ii) $A_1 + A_2 + A_3 - 2B \sim N(1.8, 0.04)$

$P[A_1 + A_2 + A_3 - 2B > 1.85] = 0.401$

(iii) $12A + 7(B_1 + B_2) \sim N(20.4, 1.685)$

$P[12A + 7(B_1 + B_2) > 21] = 0.322$

$12(A_1 + A_2 + \ldots + A_n) + 7(B_1 + B_2 + \ldots + B_{15-n}) \sim N(63 + 7.8n, 1.8375 + 1.3175n)$

$P[12(A_1 + A_2 + \ldots + A_n) + 7(B_1 + B_2 + \ldots + B_{15-n}) > 150] < 0.7.$

Largest $n = 11$