Permutations and Combinations

1(i)	YY E E S T R D A No. of arrangements = $\frac{8!}{2!} = 20160$
(ii)	YY S T R D A ↑ ↑ ↑ ↑ ↑ ↑ ↑ No. of arrangements = $6! \times \binom{7}{2} = 15120$ <u>or</u> No. of arrangements = $\frac{8!}{2!} - 7! = 15120$
2	B A R P B A P A P A **Case 1: 3 identical** No. of code words = 2 **Case 2: 2 same** No. of code words = $\binom{3}{1} \times \binom{3}{1} \times \frac{3!}{2!} = 27$ **Case 3: All different** No. of code words = ${}^4P_3 = 24$ Total number of code words = 2 + 27 +24 = 53
3(a)	AT 7 others 8 items Number of ways = $\frac{8!}{3!} \times 2!$ = 13 440
(b)	Case 1 : EEE Number of ways = 1 Case 2 : EE 1 others Number of ways = ${}^6C_1 \times \frac{3!}{2!} = 18$ Case 3 : E 2 others Number of ways = ${}^6C_2 \times 3! = 90$ Case 4 : 3 others Number of ways = ${}^6P_3 = 120$ Alternatively, combining Case 3 and Case 4: All letters are different: ${}^7P_3 = 210$ Total number of ways = 1 + 18 + 90 + 120 = 229

4(a)	No. of different 4-digit numbers = 3! x 3 = 18
(b)	*CUCUMBER* Case 1: all letters are different. No. of 4-letter code words = ${}^{6}P_{4} = 360$ Case 2: one pair of repeated letters. No. of 4-letter code words = ${}^{2}C_{1} \text{x} {}^{5}C_{2} \text{x} \frac{4!}{2!} = 240$ Case 3: two pairs of repeated letters. No. of 4-letter code words = $\frac{4!}{2!2!} = 6$ Total no. of 4-letter code words = 606
5(a)	${}^{40}C_{2} = 780$
(b)	$5! \text{ X } 2^{5} = 3840$
6(a)	No of ways = $\frac{\binom{11}{3}\binom{8}{3}\binom{5}{3}\binom{2}{2}}{3!} = 15400$
(b)	No of ways = $(8-1)! \times (4)(3) \times 2 = 120960$
7(i)	No of ways = 8!5! = 4838400
(ii)	No of ways = $\binom{7}{1} 2!5!6!2! = 2419200$
(iii)	No of ways = 7!5! = 604800
8(i)	Taking each student and his or her parents as a unit, the number of ways to arrange them and the 3 male teacher in a circle $=(5-1)!$ Each student and parents can arrange themselves in 2! ways such that the student is seated between parents. As the 3 female teachers must be separated, the number of ways to arrange them $= {}^{5}P_{3}$ Therefore, the total number of arrangements $=(5-1)!(2!)(2!)\left({}^{5}P_{3}\right)=5760$
(ii)	Taking the 3 male teachers as a unit and arranging all of them in a straight line (since seats are numbered), the number of ways to arrange all of them $=(10!)(3!)$ If the 3 male teachers are seated at seat number 1,2, 12 or 1, 11, 12, the number of ways to arrange them $=(9!)(3!)(2)$ Therefore, the total number of arrangements $=(10!)(3!)+(9!)(3!)(2)=26127360$

	Alternative: Number of arrangements in a circle without numbering = (9!)(3!) Since there are 12 people in total, therefore the total number of arrangements = (9!)(3!)(12) = 26127360
9	No. of ways the committee can be formed $= {}^4C_3\,{}^{10}C_5 + {}^4C_4\,{}^{10}C_4 = 1218$
(i)	No. of ways the committee can be seated without restriction = $(8-1)!$ = 5040
(ii)	No. of ways the committee can be seated such that no two girls are seated next to each other = 4! (5)(4)(3) = 1440 No. of ways the committee can be seated when there are two absentees = $\frac{(8-1)!}{2!}$ = 2520 or $(6-1)!\,{}^6C_2 + (7-1)! = 2520$
10(a)(i)	Number of ways = 5!x 6! = 86400
(ii)	Number of ways = C^8_2 x 2! x 2!x 7! = 564480
(b)(i)	Number of ways = 4!x 5! = 2880
(ii)	Number of ways = 4!x 3 x 4! =1728
11(i)	12! = 479001600
(ii)	$4!\times 6!\times 2 = 34560$
(iii)	$10!\times 3! = 21772800$
(iv)	$5!\times 7!\times 2 = 1209600$
12(i)	Number of ways $= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4 \times (4!)^3$ or 12! = 479001600
(ii)	Number of ways $= \left({}^6C_2 \times {}^4C_2 \times {}^2C_2\right)^2 \times (4!)^3 = 111974400$
(iii)	Number of ways $= \left({}^6C_2 \times {}^4C_2 \times {}^2C_2\right)^2 \times (2\times 2!\times 2!)^3 = 4147200$