Probability

1

8(i)	$P(Y)=\left(\frac{10}{25}\right)\left(\frac{8}{12}\right)+\left(\frac{15}{25}\right)\left(\frac{3}{10}\right)=\left(\frac{67}{150}\right)$
8(ii)	$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)=\frac{10}{25}+\frac{67}{150}-\left(\frac{10}{25}\right)\left(\frac{8}{12}\right)=\frac{29}{50}$
8(iii)	$P(X\mid Y)=\frac{P(X\cap Y)}{P(Y)}=\frac{\frac{4}{15}}{\frac{67}{150}}=\frac{40}{67}$ $P(X)=\frac{2}{5}\neq P(X\mid Y)$ Hence events X and Y are not independent.

2

8(a) (i)	$\text{P}(A)=\frac{9}{36}=\frac{1}{4}$ $\quad \text{P}(B)=\frac{4}{36}=\frac{1}{9}$ $\quad \text{P}(A\cap B)=\frac{1}{36}$ Since $\text{P}(A\cap B)=\text{P}(A)\,\text{P}(B)$, they are independent.
(ii)	$A\cup B$ represents the event the card taken is either blue or numbered 1. $\text{P}(A\cup B)=\text{P}(A)+\text{P}(B)-\text{P}(A\cap B)=\frac{9}{36}+\frac{4}{36}-\frac{1}{36}=\frac{1}{3}$
(iii)	$\text{P}(A'\mid B')=\frac{P(A'\cap B')}{P(B')}=\frac{1-\frac{1}{3}}{\frac{32}{36}}=\frac{3}{4}$
(b)	<u>Method 1</u>: Required probability $=1\times\frac{3}{35}\times\frac{2}{34}=\frac{3}{595}$ or 0.00504 <u>Method 2</u>: Required probability $=\frac{4}{36}\times\frac{3}{35}\times\frac{2}{34}\times 9=\frac{3}{595}$ <u>Method 3</u>: Required probability $=\binom{4}{3}\left(\frac{1}{36}\right)\left(\frac{1}{35}\right)\left(\frac{1}{34}\right)3!\times 9=\frac{3}{595}$ <u>Method 4</u>: Required probability $=\frac{9\binom{4}{3}}{\binom{36}{3}}=\frac{3}{595}$
(c)	<u>Method 1</u>: Required probability $=\frac{^{32}C_{20}}{^{36}C_{20}}=\frac{52}{1683}=0.030897=0.0309$

	Method 2: Required probability $= \frac{32}{36} \times \frac{31}{35} \times \frac{30}{34} \times \frac{29}{33} \times \frac{28}{32} \times ... \times \frac{17}{21} \times \frac{16}{20} \times \frac{15}{19} \times \frac{14}{18} \times \frac{13}{17}$ $= \frac{16}{36} \times \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} = 0.030897 = 0.0309$

3 (a)(i) P(A) = 18/36 = ½ (or equivalent)

P(B) = 2(5/6)(1/6) + (1/6)(1/6) = 11/36 (or equivalent)

P(C) = 3/36 = 1/12 (or equivalent)

(ii) P(B $\cap$ C) = 2/36 = 1/18

P(B)P(C) = 11/432

Since P(B $\cap$ C) $\neq$ P(B)P(C), so B and C are not independent.

(iii) P(C'|A) = P(A $\cap$ C')/P(A)

= (15/36) $\div$ (1/2) = 5/6 (or equivalent)

(b)(i) P(call for Daisy and Daisy is at home)

= (0.2)(0.75)

= 0.15

(ii) P(person to whom call is made is at home)

= 0.15 + (0.3)(0.5) + (0.5)(0.8)

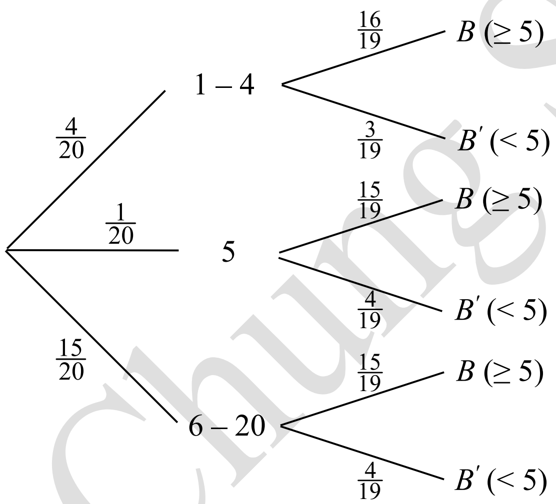
= 0.70

4

(a) (i)	P(all three cards even) $= \frac{10}{20}\frac{9}{19}\frac{8}{18} = \frac{2}{19}$ or $= \frac{\binom{10}{3}}{\binom{20}{3}} = \frac{2}{19}$
(ii)	P(exactly one even) $= \frac{10}{20}\frac{10}{19}\frac{9}{18} \times \binom{3}{1} = \frac{15}{38}$ or $= \frac{\binom{10}{1}\binom{10}{2}}{\binom{20}{3}} = \frac{15}{38}$
(b)	$\mathrm{P}(B) = \frac{4}{20}\frac{16}{19} + \frac{16}{20}\frac{15}{19} = \frac{4}{5}$

$$\mathrm{P}(A \cap B) = \mathrm{P}(\text{1st card} \le 5 \ \ \& \ \ \text{2nd card} \ge 5)$$
$$= \mathrm{P}(\text{1st card} = 5 \ \ \& \ \ \text{2nd card} > 5) + \mathrm{P}(\text{1st card} < 5 \ \ \& \ \ \text{2nd card} \ge 5)$$
$$= \frac{1}{20}\frac{15}{19} + \frac{4}{20}\frac{16}{19}$$
$$= \frac{79}{380}$$
$$\mathrm{P}\left(B \middle| A\right) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)} = \frac{\frac{79}{380}}{\frac{1}{4}} = \frac{79}{95}$$

<u>Using tree diagram</u>:



(i)

(ii)

$$\mathrm{P}\left(B\right) = \frac{4}{20}\frac{16}{19} + \frac{1}{20}\frac{15}{19} + \frac{15}{20}\frac{15}{19} = \frac{4}{5}$$

(iii)

$$\mathrm{P}(A \cap B) = \frac{4}{20}\frac{16}{19} + \frac{1}{20}\frac{15}{19} = \frac{79}{380}$$
$$\mathrm{P}\left(B \middle| A\right) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)} = \frac{\frac{79}{380}}{\frac{1}{4}} = \frac{79}{95}$$

5

6(a) (i)	Since A and B are mutually exclusive, $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ $\Rightarrow \mathrm{P}(B) = \mathrm{P}(A \cup B) - \mathrm{P}(A) = 0.6 - 0.2 = 0.4$ □
6(a) (ii)	Since A and B are independent, $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.2\,\mathrm{P}(B)$ … (1) Also, $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\Rightarrow \mathrm{P}(B) = \mathrm{P}(A \cup B) - \mathrm{P}(A) + \mathrm{P}(A \cap B)$ $\Rightarrow \mathrm{P}(B) = 0.6 - 0.2 + 0.2\mathrm{P}(B)$ [from (1)] $\Rightarrow 0.8\mathrm{P}(B) = 0.4 \Rightarrow \mathrm{P}(B) = 0.5$ □
6(b) (i)	Required probability $= (0.1 \times 0.01) + (0.6 \times 0.15) + (0.3 \times 0.25)$ $= 0.166$ □
6(b) (ii)	$\mathrm{P}(\text{good risks} \mid \text{no accidents}) = \dfrac{\mathrm{P}(\text{good risks and no accidents})}{\mathrm{P}(\text{no accidents})}$ $= \dfrac{0.1 \times (1 - 0.01)}{1 - 0.166}$ [from **(i)**] $= \dfrac{0.099}{0.834} = \dfrac{33}{278}$ or 0.119 (3 s.f.)

6

Solution

(a) P(Kelvin is the first to pick a prime | game ends on the second draw)

$$=\frac{\text{P}\left(\text{Kelvin picks a prime on his second draw}\right)}{\text{P}\left(\text{Game ends on the second draw}\right)}$$

$$=\frac{\text{P}\left(\text{Kelvin picks a prime on his second draw}\right)}{\text{P}\left(\text{Kelvin picks a prime on his second draw}\right)+\text{P}\left(\text{Raju picks a prime on his second draw}\right)}$$

$$=\frac{\frac{12}{20}\times\frac{11}{19}\times\frac{8}{18}}{\left(\frac{12}{20}\times\frac{11}{19}\times\frac{8}{18}\right)+\left(\frac{12}{20}\times\frac{11}{19}\times\frac{10}{18}\times\frac{8}{17}\right)}$$

$$=\frac{17}{27}\text{ (or 0.630)}$$

(b) P(Raju picks a prime first) =

$$\frac{3}{5}\left(\frac{2}{5}\right)+\frac{3}{5}\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)+\frac{3}{5}\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)+\ldots$$

$$=\frac{6}{25}+\frac{6}{25}\left(\frac{9}{25}\right)+\frac{6}{25}\left(\frac{9}{25}\right)^2+\ldots$$

$$=\frac{\frac{6}{25}}{1-\frac{9}{25}}$$

$$==\frac{3}{8}$$

7 (a) $P(A \mid B') = \frac{P(A\cap B')}{P(B')} = \frac{2}{3}$ $\qquad$ $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{2}{7}$

$P(A \cap B') = \frac{2}{3}P(B') = \frac{2}{3}(1 - P(B))$ $\qquad$ $P(A \cap B) = \frac{2}{7}P(B)$

$P(A) = P(A \cap B') + P(A \cap B) = \frac{2}{3}(1 - P(B)) + \frac{2}{7}P(B)$

$\frac{1}{2} = \frac{2}{3} - \frac{8}{21}P(B) \quad \Rightarrow \quad P(B) = \frac{7}{16}$

A and B are not independent as $P(A \mid B) \neq P(A)$

(b)(i) $\qquad$ Probability $= 1(\frac{1}{15}) + \left(\frac{14}{15}\right)\left(\frac{1}{15}\right) + \left(\frac{14}{15}\right)^2\left(\frac{1}{15}\right) = \frac{631}{3375}$

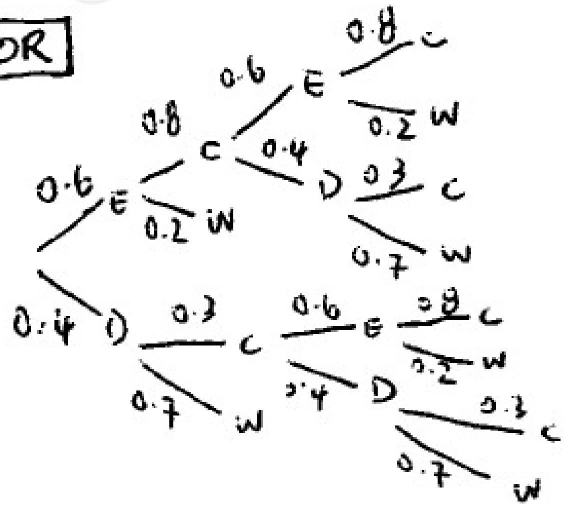
(ii) Probability $= \frac{631}{3375} + \left(\frac{14}{15}\right)^3\left(\frac{1}{15}\right) = \frac{12209}{50625}$

8

(i) Prob $= 0.6^3 = 0.216$.

(ii) Prob $= (0.6\times0.8 + 0.4\times0.3)^2 = 0.36$

OR



prob $= 0.6\times0.8\times(0.6\times0.8+0.4\times0.3)$

$\quad + 0.4\times0.3(0.6\times0.8+0.4\times0.3) = 0.36$

(iii) $\dfrac{P(\text{a challenging question and gave correct ans})}{P(\text{correct ans.})}$

$= \dfrac{0.4\times0.3}{0.6\times0.8+0.4\times0.3} = 0.2$