2015 'A' Level H1 Maths Solutions

Note Title 25/11/2015

1. Quadratic Functions

$2k + (k-4)x - 2x^2$ is always negative

$\Rightarrow -2x^2 + (k-4)x + 2k < 0$

$\Rightarrow D = (k-4)^2 - 4(-2)(2k) < 0$

$\Rightarrow k^2 + 8k + 16 < 0$

$\Rightarrow (k+4)^2 < 0$

$\Rightarrow$ no real solutions for k

$\therefore$ there are no real values of k for $2k + (k-4)x - 2x^2 < 0$

2. Differentiation and Integration Techniques

(i) $\frac{d}{dx}\left(\frac{3}{(2x-1)^4}\right) = \frac{d}{dx}\left[3(2x-1)^{-4}\right]$

$= -12\,(2x-1)^{-5}\,(2)$

$= \frac{-24}{(2x-1)^5}$

(ii) $\int_{\frac{1}{2}}^{1} \left(x + \frac{2}{x}\right)^2 dx = \int_{\frac{1}{2}}^{1} \left(x^2 + 4 + 4x^{-2}\right) dx$

$= \left[\frac{x^3}{3} + 4x + \frac{4x^{-1}}{-1}\right]_{\frac{1}{2}}^{1}$

$= \left(\frac{1}{3} + 4 - 4\right) - \left(\frac{1}{24} + 2 - 8\right)$

$= \frac{1}{3} - \left(-\frac{143}{24}\right)$

$= \frac{151}{24}$

3 Applications of Differentiation and Integration

$$y = 12x + 8e^{-2x}$$

(i) $\frac{dy}{dx} = 12 - 16e^{-2x}$

At stationary pt, $\frac{dy}{dx} = 0$

$$12 - 16e^{-2x} = 0$$

$$e^{-2x} = \frac{16}{12}$$

$$-2x = \ln \frac{4}{3}$$

$$x = -\frac{1}{2} \ln \frac{4}{3}$$

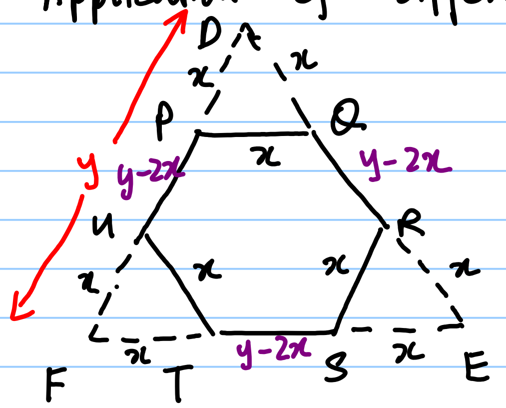
(ii) Required Area $= \int_0^a 12x + 8e^{-2x} \, dx$

$$= \left[\frac{12x^2}{2} + \frac{8e^{-2x}}{-2} \right]_0^a$$

$$= (6a^2 - 4e^{-2a}) - (-4)$$

$$= 6a^2 - 4e^{-2a} + 4$$

4. Application of Differentiation



Perimeter of PQRSTU = 30

$3(y-2x) + 3x = 30$

$3(y-x) = 30 \Rightarrow y = 10+x$

Area of PQRSTU, A

$= \text{Area of } \Delta DEF - 3(\text{Area of } \Delta DPQ)$

$= \frac{1}{2}y^2 \sin 60^\circ - 3\left(\frac{1}{2}x^2 \sin 60^\circ\right)$

$= \frac{1}{2}y^2\left(\frac{\sqrt{3}}{2}\right) - \frac{3}{2}x^2\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4}(y^2 - 3x^2)$

$= \frac{\sqrt{3}}{4}\left[(10+x)^2 - 3x^2\right]$

$= \frac{\sqrt{3}}{4}\left[100 + 20x + x^2 - 3x^2\right]$

$= \frac{\sqrt{3}}{2}\left[50 + 10x - x^2\right]$ (shown)

$\frac{dA}{dx} = \frac{\sqrt{3}}{2}(10-2x)$

When A is max, $\frac{dA}{dx} = 0$

$\frac{\sqrt{3}}{2}(10-2x) = 0$

$x = 5$

$\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2}(-2) < 0$

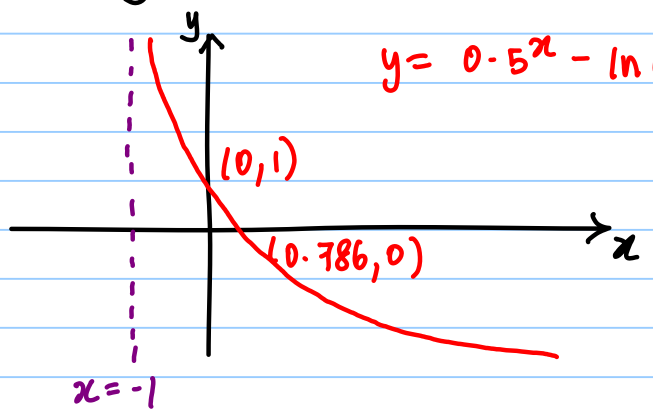
$\Rightarrow$ A is max when $x = 5$

$\therefore$ max $A = \frac{\sqrt{3}}{2}(50+50-25)$

$= \frac{75\sqrt{3}}{2}$ cm^2

5. Graphing Techniques / Application of Differentiation

(i)



(ii) Using GC, when $x = 0.5$, $\frac{dy}{dx} = -1.156796$

≈ -1.157 (3 d.p.)

(iii) When $x = 0.5$, $y = 0.30164 \Rightarrow P = (0.5, 0.30164)$

Equation of normal at P is

$$y - 0.30164 = -\frac{1}{1.156796}(x - 0.5)$$

When $y = 0$, $x = 0.151064 \Rightarrow A = (0.151064, 0)$

When $x = 0$, $y = -0.130588 \Rightarrow B = (0, -0.130588)$

$\therefore AB = \sqrt{(0.151064)^2 + (0.130588)^2}$

$= 0.200$

6. Normal Distribution

Let X be the mass of peaches sold

$X \sim N(\mu, \sigma^2)$

$P(X < 40) = 0.2 \Rightarrow P\left(Z < \frac{40-\mu}{\sigma}\right) = 0.2$

$\Rightarrow \frac{40-\mu}{\sigma} = -0.84162$

$\mu - 0.84162\sigma = 40$ —— (1)

$P(X > 60) = 0.25 \Rightarrow P\left(Z > \frac{60-\mu}{\sigma}\right) = 0.25$

$\Rightarrow P\left(Z < \frac{60-\mu}{\sigma}\right) = 0.75$

$\Rightarrow \frac{60-\mu}{\sigma} = 0.67449$

$\mu + 0.67449\sigma = 60$ —— (2)

Using GC, $\mu = 51.1$, $\sigma = 13.19165$

$\Rightarrow \sigma^2 = 174$

7. Sampling Methods

(i) Number the students from 1 to 1200
Determine the sampling interval $k = \frac{1200}{100} = 12$
Select at random the 1st student from 1st 12 students in the list.
Select every 12th student thereafter.

(ii) Disadvantage of systematic sample :
Sample might not be representative as certain group such as year one students might not be included in the sample.

(iii) Stratified sample might be more appropriate.

8. Probability

$P(A) = p$, $P(B) = 2p$, $P(A \cup B) = 0.42$, $P(A \cap B) = 0.03$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.42 = p + 2p - 0.03$

$3p = 0.42 + 0.03 = 0.45$

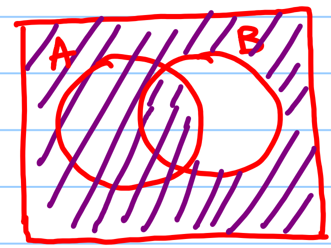
$\therefore p = 0.15$ (shown)

(ii) $P(A \cup B') = 1 - P(A' \cap B)$

$= 1 - [P(B) - P(A \cap B)]$

$= 1 - [2(0.15) - 0.03]$

$= 0.73$



(iv) $P(A) \cdot P(B) = 0.15(0.3) = 0.045$

$P(A \cap B) = 0.03$

$\because P(A) \cdot P(B) \neq P(A \cap B)$, A & B are not independent

$\Rightarrow \therefore$ A & B' are not independent

9. Binomial Distribution and Approximation

Let X be no of sixes obtained out of 8 times

$X \sim B(8, \frac{1}{6})$

(i) $P(X=3) = 0.104$

(ii) $P(X<4) = P(X \leq 3) = 0.969$

(iii) Let y be no of sixes obtained out of 600 times

$y \sim B(600, \frac{1}{6})$

$\because n = 600$ is large, $np = 600(\frac{1}{6}) = 100 > 5$

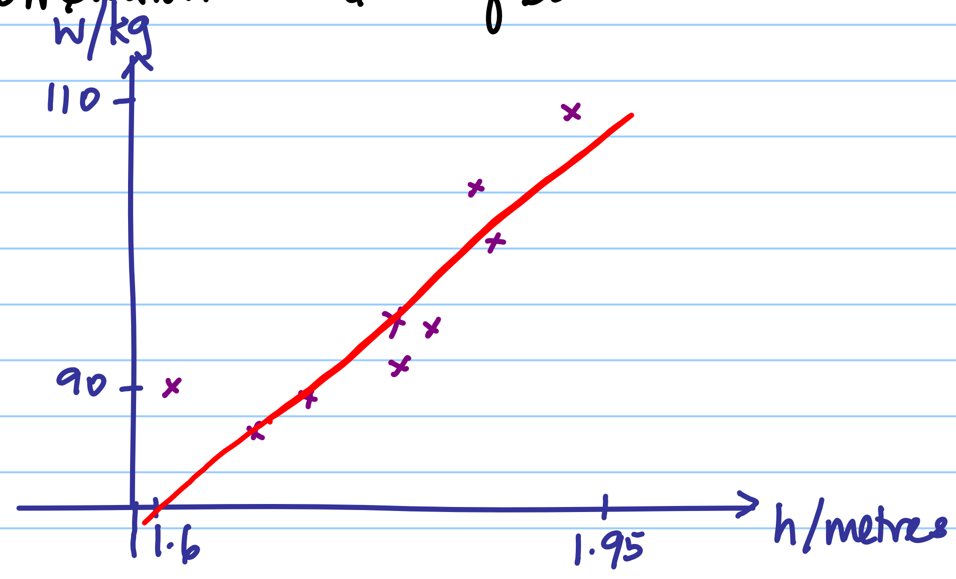
$nq = 600(\frac{5}{6}) = 500 > 5$

$\therefore y \sim N(100, \frac{250}{3})$ approx.

$P(90 \leq y \leq 100) \overset{c.c.}{\cong} P(89.5 \leq y \leq 100.5)$

$= 0.397$

10. Correlation and Regression



(ii) Using GC, $r = 0.922$.
$\because$ r is close to 1, there is a strong positive linear correlation between the height of member and the weight of rowing club.

(iii) Using GC, $w = -6.3005 + 58.033h$
$w = -6.30 + 58.0h$

(iv) When $h = 1.66$, $w = -6.3005 + 58.033(1.66)$
$= 90.0$ kg

This estimate is reliable as $h = 1.66$ lies within the given data range and $r = 0.922$ is close to 1.

11. Normal Distribution

Let M and W be the masses of man and women respectively.

$M \sim N(77, 9.8^2)$, $W \sim N(62, 10.6^2)$

(i) $P(-2 < M - 77 < 2) = P(75 < M < 79)$
$= 0.162$

(ii) Let $X = M_1 + M_2 + M_2 - (W_1 + W_2 + W_3 + W_4) \sim N(-17, 737.56)$

$E(X) = 3(77) - 4(62) = -17$

$var(X) = 3(9.8^2) + 4(10.6^2) = 737.56$

$P(M_1 + M_2 + M_2 > W_1 + W_2 + W_3 + W_4) = P(X > 0)$
$= 0.266$

(iii) Let $y = M_1 + M_2 + M_2 + W_1 + W_2 + W_3 + W_4 \sim N(479, 737.56)$

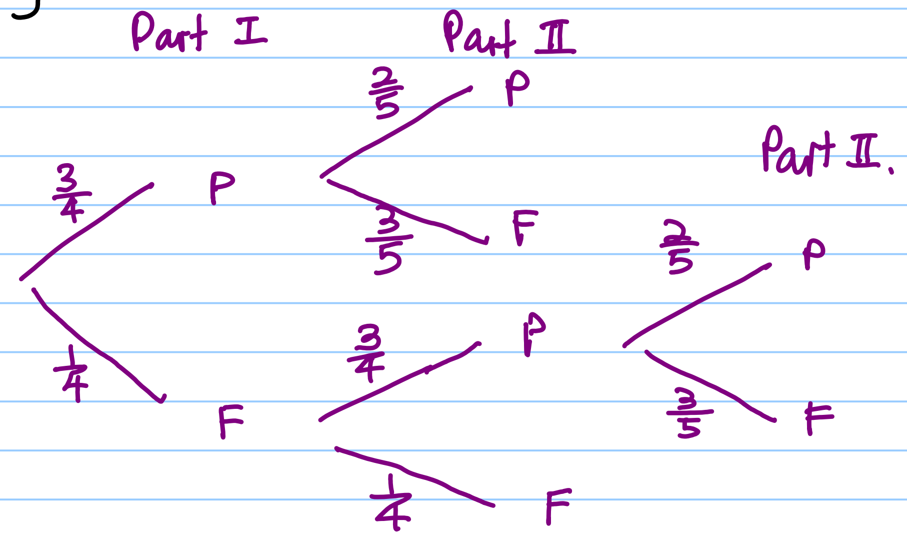
$E(y) = 3(77) + 4(62) = 479$

$var(y) = 3(9.8^2) + 4(10.6^2) = 737.56$

P(they can safely travel in lift) $= P(y \leq 460)$
$= 0.242$

12. Probability

(i)



(ii) $P(\text{ student succeed}) = \frac{3}{4} \times \frac{2}{5} + \frac{1}{4} \times \frac{3}{4} \times \frac{2}{5} = \frac{3}{8}$

(iii) $P(\text{ student succeed} \mid \text{fails Part I in 1st attempt})$

$= \dfrac{P(\text{ Student succeed} \cap \text{fails Part I in 1st attempt})}{P(\text{fails Part I in 1st attempt})}$

$= \dfrac{\frac{1}{4} \times \frac{3}{4} \times \frac{2}{5}}{\frac{1}{4}} = \frac{3}{10}$

(iv) Let X be no. of students who will be successful out of 5.

$X \sim B(5, \frac{3}{8})$

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 0.619$

13. Hypothesis Testing

Let X be length of fish in lake

To test $H_0: \mu = 15.2$
against $H_1: \mu \neq 15.2$

$X \sim N(15.2, 2.1^2)$
$\bar{X} \sim N\left(15.2, \frac{2.1^2}{30}\right)$

Test statistic, $Z = \dfrac{\bar{X} - 15.2}{2.1/\sqrt{30}} \sim N(0,1)$

Reject H_0 if $p < \alpha = 0.05$

Using GC, $p = 0.0679$, $z = -1.83$

$\because$ $p = 0.0679 > 0.05$, we do not reject H_0 and conclude that there is insufficient evidence at 5% level of significance the mean length of fish is not 15.2 ie. the scientist's claim should not be rejected.

(ii) $\sum(x - 18) = 32$, $\sum(x-18)^2 = 325$, $n = 40$

Unbiased estimate of $\mu = \bar{x} = \frac{32}{40} + 18 = 17.2$

" " " $\sigma^2 = s^2 = \frac{1}{39}\left[325 + \frac{32^2}{40}\right]$

$= \frac{499}{65}$

(iii) Unbiased estimate is an estimate whose expected value is equal to the population parameter being estimated.

(iv) To test $H_0: \mu = 18$
against $H_1: \mu < 18$

$\because$ $n = 40$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(18, \frac{s^2}{40}\right)$ approx.

Test Statistic, $Z = \dfrac{\bar{X} - 18}{s/\sqrt{40}} \sim N(0,1)$

Using GC, $p = 0.0339$, $z = -1.83$

$\because$ H_0 is rejected, $p < \alpha \Rightarrow \alpha > 0.033917$
$\Rightarrow \alpha\% > 0.033914 \times 100\% = 3.3914$

$\therefore$ Set of values of $\alpha = \{\alpha \in \mathbb{R}, 3.40 \leq \alpha \leq 100\}$