GCE A Level H1 Maths 2016

1. Differentiation Techniques

(i) $\frac{d}{dx}\left[2\ln(3x^2+4)\right] = 2\left[\frac{1}{3x^2+4}(6x)\right]$

$= \frac{12x}{3x^2+4}$ #

(ii) $\frac{d}{dx}\left[\frac{1}{2(1-3x)^2}\right] = \frac{d}{dx}\left[\frac{1}{2}(1-3x)^{-2}\right]$

$= \frac{1}{2}\left[-2(1-3x)^{-3}(-3)\right] = \frac{3}{(1-3x)^3}$ #

2. Logarithmic & Exponential Functions

$2e^{2x} \geqslant 9 - 3e^{x}$, $u = e^{x}$

$2u^2 + 3u - 9 \geqslant 0$

$(u+3)(2u-3) \geqslant 0$

$+ \quad - \quad +$; $-3 \quad \frac{3}{2}$

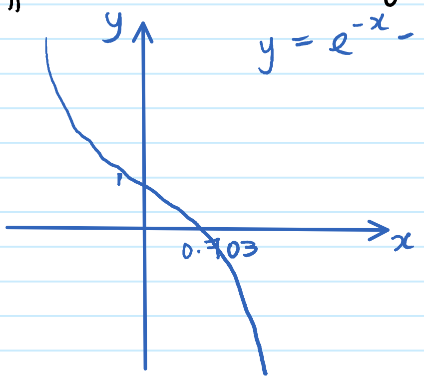
$u \leq -3$ or $u \geqslant \frac{3}{2}$

$e^{x} \leq -3$ or $e^{x} \geqslant \frac{3}{2} \Rightarrow x \geqslant \ln\frac{3}{2}$ #

(rejected $\because e^{x} > 0$)

3. Differentiation & Integration

(i)



(ii) $\frac{d}{dx}\left(e^{-x} - x^2\right)\Big|_{x=0.5}$

$= -1.60653$

≈ -1.61 #

(iii) When $x = 0.5$, $y = e^{-0.5} - 0.5^2 = 0.35653$

Equation of normal at $x = 0.5$ is

$y - 0.35653 = -\frac{1}{(-1.60653)}(x - 0.5)$

$y = 0.6224595x + 0.0453002$

$\therefore y = 0.622x + 0.0453$ #

(iv) $\int_0^k (e^{-x} - x^2)\,dx = \left[-e^{-x} - \frac{x^3}{3}\right]_0^k$

$= \left(-e^{-k} - \frac{k^3}{3}\right) - (-1 - 0) = -e^{-k} - \frac{k^3}{3} + 1$ #

4. Applications of Differentiation & Integration

(i)

$$y = 1 + 6x - 3x^2 - 4x^3$$

$$\frac{dy}{dx} = 6 - 6x - 12x^2$$

At stationary points, $\frac{dy}{dx} = -6(2x^2 + x - 1) = 0$

$$(2x-1)(x+1) = 0$$

$$x = \frac{1}{2} \text{ or } x = -1$$

When $x = \frac{1}{2}$, $y = 1 + 3 - \frac{3}{4} - \frac{1}{2} = 2.75$

" $x = -1$, $y = 1 - 6 - 3 + 4 = -4$

$\therefore$ Coordinates of stationary pts are $(\frac{1}{2}, 2.75)$ & $(-1, -4)$ #

(ii)

x	$(\frac{1}{2})^-$	$(\frac{1}{2})$	$(\frac{1}{2})^+$
$\frac{dy}{dx}$	+	0	−

OR

$\frac{d^2y}{dx^2} = -6 - 24x$

When $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = -18 < 0$

$\Longrightarrow \therefore (\frac{1}{2}, 2.75)$ is a max. pt #

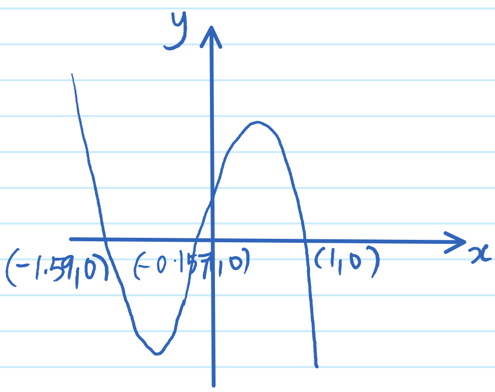
x	-1^-	-1	-1^+
$\frac{dy}{dx}$	−	0	+

OR

When $x = -1$, $\frac{d^2y}{dx^2} = 18 > 0$

$\Longrightarrow \therefore (-1, -4)$ is a min. pt #

(iii)



(iv) Area $= \int_{0.5}^{1} 1 + 6x - 3x^2 - 4x^3 \, dx$

$= 0.9375$ #

5. Solving Equations

(a) Area of V-shape ABEDFCA $= 2\sqrt{3}$

$$\frac{1}{2}(2x)^2\sin 60^\circ - \frac{1}{2}y^2\sin 60^\circ = 2\sqrt{3}$$

$$\sqrt{3}x^2 - \frac{\sqrt{3}}{4}y^2 = 2\sqrt{3}$$

$$x^2 - \frac{y^2}{4} = 2$$

$$\therefore 4x^2 - y^2 = 8 \ \# \text{ (shown)} \quad \text{---(1)}$$

(ii) Perimeter of ABEDFCA $= 10$

$$2(2x) + 2y + (2x - y) = 10$$

$$6x + y = 10 \Rightarrow y = 10 - 6x \quad \text{---(2)}$$

Sub (2) into (1): $4x^2 - (10-6x)^2 = 8$

$$4x^2 - (100 - 120x + 36x^2) = 8$$

$$-32x^2 + 120x - 108 = 0$$

$$x = 2.25 \text{ or } x = 1.5$$

$$y = -3.5 \text{ or } y = 1$$

$$\because y > 0, \quad x = 1.5 \ \& \ y = 1 \ \#$$

6. Sampling

(i) No of male surveyed $= \frac{1260}{2400} \times 80 = 42$

" " female " $= \frac{1140}{2400} \times 80 = 38$

The manager surveyed at random 42 males students & 38 female students to obtain a stratified sample.

(ii) Stratified sampling could give a more representative sample of how much money students spend on music each year compared to simple random sampling.

(iii) Unbiased estimate for population mean $= \bar{x}$

$$= \frac{\Sigma x}{n} = \frac{312}{80} = 3.9 \ \#$$

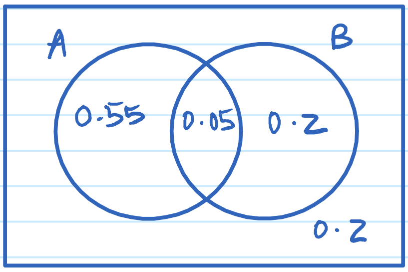
Unbiased estimate for population variance $= s^2$

$$= \frac{1}{n-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right] = \frac{1}{79}\left[1328 - \frac{312^2}{80}\right]$$

$$= 1.40759 \approx 1.41 \ \#$$

7. Probability

(i)



(i)(a) $P(\geq 1 \text{ A \& B}) = P(A \cup B)$

$= 0.55 + 0.05 + 0.2$

$= 0.8$ #

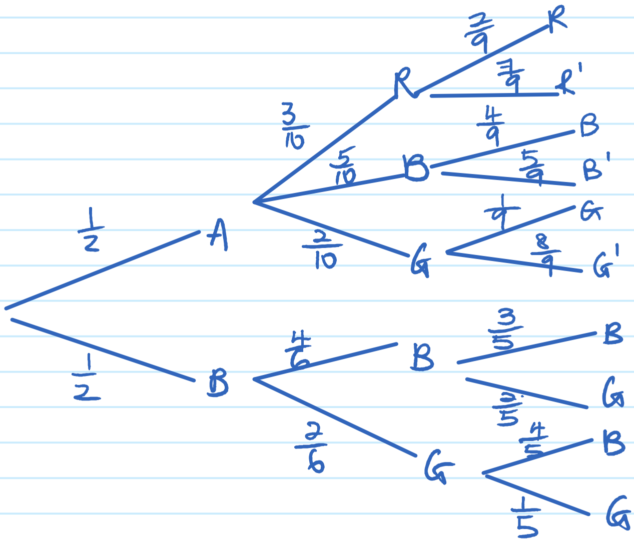
(b) $P(\text{exactly 1 of A \& B})$

$= P(A \cup B) - P(A \cap B)$

$= 0.8 - 0.05 = 0.75$ #

(ii) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{0.55}{1 - 0.25} = \dfrac{11}{55}$ #

8. Probability



(i) $P(\text{both R}) = \frac{1}{2}\left(\frac{3}{10}\right)\left(\frac{2}{9}\right) = \frac{1}{30}$ #

(ii) $P(\text{different colours}) = \frac{1}{2}\left[\frac{3}{10}\left(\frac{7}{9}\right) + \frac{5}{10}\left(\frac{5}{9}\right) + \frac{2}{10}\left(\frac{8}{9}\right)\right] + \frac{1}{2}\left[\frac{4}{6}\left(\frac{2}{5}\right) + \frac{2}{6}\left(\frac{4}{5}\right)\right]$

$= \frac{1}{2}\left(\frac{62}{90}\right) + \frac{1}{2}\left(\frac{8}{15}\right) = \frac{11}{80}$ #

$P(\text{different colours}) = 1 - P(\text{both R}) - P(\text{both B}) - P(\text{both G})$

$= 1 - \frac{1}{30} - \frac{1}{2}\left[\left(\frac{5}{10}\right)\left(\frac{4}{9}\right) + \left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\right] - \frac{1}{2}\left[\frac{2}{10}\left(\frac{1}{9}\right) + \frac{2}{6}\left(\frac{1}{5}\right)\right]$

$= 1 - \frac{1}{30} - \frac{14}{45} - \frac{2}{45}$

$= \frac{11}{18}$ #

(iii) $P(\text{both R} \mid \text{same colour}) = \dfrac{P(\text{both R})}{P(\text{same colour})}$

$= \dfrac{\frac{1}{30}}{1 - \frac{11}{18}} = \dfrac{3}{35}$ #

9. Binomial Distribution

(i) Let X be no. of batteries with lifetime < 2 yrs out of 8 batteries.

$X \sim B(8, 0.6)$

(a) $P(X = 8) = 0.016796 \approx 0.0168$ #

(b) $P(X \geq 4) = 1 - P(X \leq 3) = 0.8263296 \approx 0.826$ #

(ii) Let y be no. of packs with $\geq$ half of batteries < 2 yrs out of 4 packs.

$y \sim B(4, 0.8263296)$

$P(y \leq 2) = 0.1417924 \approx 0.142$ #

(iii) Let W be no. of batteries with lifetime < 2 yrs out of 10×8 batteries

$W \sim B(80, 0.6)$

$\because n = 80$ is large, $np = 80(0.6) = 48 > 5$, $n(1-p) = 80(0.4) = 32 > 5$,

$\therefore W \sim N(48, 19.2)$ approx.

$P(W \geq 40) \overset{c.c.}{=} P(W \geq 39.5) = 0.97380115 \approx 0.974$ #

10. Hypothesis Testing

(i) Let X be the top speed of cheetahs

$H_0: \mu = 95$

$H_1: \mu \neq 95$

Test statistic, $Z = \dfrac{\bar{X} - 95}{4.1/\sqrt{40}} \sim N(0,1)$

Using GC, $p = 0.0449258$, $z = 2.00534\,68$

$\because p = 0.0449 < 0.05$, we reject H_0 and conclude that there's sufficient evidence at 5% level of significance that $\mu \neq 95$ ie. the scientist's claim is incorrect.

(ii) $H_0: \mu = 95$

$H_1: \mu > 95$

Mean top speed not $> 95 \Rightarrow$ Do not reject H_0

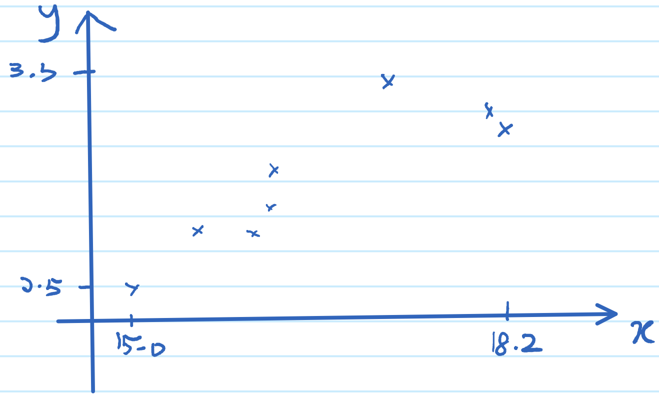
$\Rightarrow z_{calc} < z_{crit}$

$\Rightarrow \dfrac{\bar{x} - 95}{4.1/\sqrt{40}} < 1.6448536 \quad \therefore \bar{x} < 96.0663$

$\therefore$ Set of values of $\bar{x} = \{x \in \mathbb{R}^+ : \bar{x} < 96.0\}$ #

11. Correlation and Regression

(i)



(ii) Using GC,

$r = 0.9030227 \approx 0.903$ #

(iii) $y = -1.88739 + 0.293668x$

$\approx -1.89 + 0.294x$

(iv) When $x = 16.9$, $y = 3.0756$

$\therefore$ Time taken to run 1000 m ≈ 3.08 mins #

Estimate is reliable since $x = 16.9$ lies within the given data range and $r = 0.903$ is close to 1.

(v) Using GC, $r = 0.5682278566 \approx 0.568$ #

(vi) Answer to part (ii) is more likely to represent the correlation $\because$ $r = 0.903$ shows a stronger positive linear correlation between x and y as it is closer to 1 than $r = 0.568$

12. Normal Distribution

Let X & Y be the masses of individual biscuits & empty boxes.

$X \sim N(20, 1.1^2)$, $Y \sim N(5, 0.8^2)$

(i) $P(X < 19) = 0.181651 \approx 0.182$ #

(ii) Let $T = X_1 + X_2 + \ldots + X_{12} + Y$

$E(T) = 12(20) + 5 = 245$

$var(T) = 12(1.1^2) + 0.8^2 = 15.16$ $\Rightarrow T \sim N(245, 15.16)$

$P(T > 248) = 0.220502 \approx 0.221$ #

(iii) Let $C = 0.6(X_1 + X_2 + \ldots + X_{12}) + 0.2Y$

$E(C) = 0.6(12 \times 20) + 0.2(5) = 145$

$var(C) = 0.6^2(12 \times 1.1^2) + 0.2^2(0.8^2) = 5.2528$

$C \sim N(145, 5.2528)$

$P(142 < C < 149) = 0.864257 \approx 0.864$ #