GCE A Level 2017 Paper 2

1. Application of Differentiation – Parametric Equations and Equation of Tangent

(i) Sub $x = \frac{3}{t}$, $y = 2t$ into $y = 2x$:

$$2t = 2\left(\frac{3}{t}\right) \Rightarrow t^2 = 3$$
$$t = \pm\sqrt{3}$$

When $t = \sqrt{3}$, $x = \sqrt{3}$, $y = 2\sqrt{3} \Rightarrow A = (\sqrt{3}, 2\sqrt{3})$

When $t = -\sqrt{3}$, $x = -\sqrt{3}$, $y = -2\sqrt{3} \Rightarrow B = (-\sqrt{3}, -2\sqrt{3})$

Length of $AB = \sqrt{(\sqrt{3}+\sqrt{3})^2 + (2\sqrt{3}+2\sqrt{3})^2}$
$$= \sqrt{36+48} = \sqrt{84} = 2\sqrt{21}$$

(ii) $\frac{dx}{dt} = -\frac{3}{t^2}$, $\frac{dy}{dt} = 2$

$$\frac{dy}{dx} = \frac{2}{\left(-\frac{3}{t^2}\right)} = -\frac{2t^2}{3}$$

When $t = p$, $\frac{dy}{dx} = -\frac{2p^2}{3}$

Equation of tangent at P is $y - 2p = -\frac{2p^2}{3}\left(x - \frac{3}{p}\right)$

When $y = 0$, $-2p = -\frac{2p^2}{3}x + 2p$

$$\Rightarrow x = -4p \times -\frac{3}{2p^2} = \frac{6}{p} \Rightarrow D = \left(\frac{6}{p}, 0\right)$$

When $x = 0$, $y = 2p + 2p = 4p \Rightarrow E = (0, 4p)$

$$F = \left(\frac{\frac{6}{p}+0}{2}, \frac{4p}{2}\right) = \left(\frac{3}{p}, 2p\right)$$

$x = \frac{3}{p} \Rightarrow p = \frac{3}{x}$

$y = 2p \Rightarrow y = 2\left(\frac{3}{x}\right) \quad \therefore xy = 6$

2. AP / GP

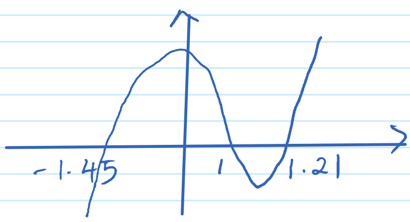
AP : $a = 3$, $S_{13} = 156$

$$\frac{13}{2}\left[2(3) + 12d\right] = 156 \Rightarrow d = 1.5$$

GP : $a = 3$, $S_{13} = 156$

$$\frac{3(1-r^{13})}{1-r} = 156$$
$$1 - r^{13} = 52(1-r) = 52 - 52r$$
$$\therefore r^{13} - 52r + 51 = 0 \quad \text{(shown)}$$

Using GC, $r = 1$ or -1.45 or 1.21



$r \neq 1$ $\because$ all terms of GP are equal if $r = 1$.

$\therefore r = 1.21$ or -1.45.

(ii) T_n of GP $>$ $100 \times T_n$ of AP

$3(1.210024)^{n-1} > 100\left[3 + (n-1)(1.5)\right]$

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$$3(1.210024)^{n-1} > 100\left[3+(n-1)(1.5)\right]$$

Using GC:

n	$3(1.210024)^{n-1}$	$100[3+(n-1)(1.5)]$
41	6150.1	6300
42	7441.7	6450
43	9004.7	6600

$\Rightarrow n \geqslant 42$

$\therefore$ Smallest $n = 42$

3. Transformation of graphs / Functions

(a)

	Pts where curve cuts x-axis	y-axis
$y=f(x)$	$(a,0)$	$(0,b)$
(i) $y=f(2x)$	$(\frac{a}{2},0)$	$(0,b)$
(ii) $y=f(x-1)$	$(a+1,0)$	$-$
(iii) $y=f(2x-1)$	$(\frac{a+1}{2},0)$	$-$
(iv) $y=f^{-1}(x)$	$(b,0)$	$(0,a)$

(b) $g: x \longrightarrow 1-\frac{1}{1-x}$, $x\in\mathbb{R}$, $x\neq a$

(i) $a=1$. This value has to be excluded from D_g as g is undefined at $x=1$.

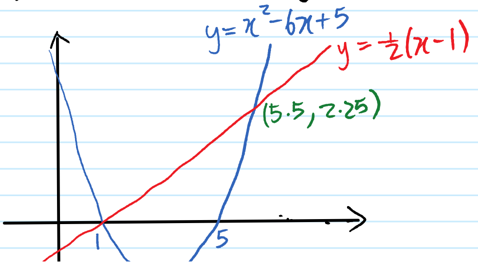
(ii)
$$g^2(x) = g\left(1-\frac{1}{1-x}\right)$$
$$= 1-\frac{1}{1-\left(1-\frac{1}{1-x}\right)}$$
$$= 1-\frac{1}{\frac{1}{1-x}} = 1-(1-x) = x$$

$$g^2(x) = x \Rightarrow g^{-1}(x) = g(x)$$
$$= 1-\frac{1}{1-x}$$

(iii)
$$g^2(b) = g^{-1}(b) \Rightarrow b = 1-\frac{1}{1-b}$$
$$\frac{1}{1-b} = 1-b$$
$$(1-b)^2 = 1$$
$$1-2b+b^2 = 1$$
$$b(b-2) = 0$$
$$b=0 \text{ or } b=2$$

4. Applications of Integration — Area + Volume

(a)



$$A = \int_1^{5.5} \tfrac{1}{2}(x-1) - (x^2-6x+5)\, dx$$
$$= 15.1875$$
$$\approx 15.2 \text{ unit}^2$$

≈ 15.2 unit^2

(b) $x = \frac{\sqrt{y}}{a-y^2} \Rightarrow V = \pi \int_0^1 \left(\frac{\sqrt{y}}{a-y^2}\right)^2 dy$

$= \pi \int_0^1 \frac{y}{(a-y^2)^2}\, dy$

$= \frac{\pi}{(-2)} \int_0^1 -2y\,(a-y^2)^{-2}\, dy$

$= -\frac{\pi}{2}\left[\frac{(a-y^2)^{-1}}{-1}\right]_0^1 = \frac{\pi}{2}\left[\frac{1}{(a-y^2)}\right]_0^1$

$= \frac{\pi}{2}\left[\frac{1}{a-1} - \frac{1}{a}\right]$

$= \frac{\pi}{2}\left[\frac{a-(a-1)}{a(a-1)}\right] = \frac{\pi}{2a(a-1)}$

(ii) $V_2 = \frac{\cancel{\pi}}{\cancel{2}b(b-1)} = 4\left(\frac{\cancel{\pi}}{\cancel{2}a(a-1)}\right)$

$\frac{1}{4}a(a-1) = b(b-1) = b^2 - b$

$= \left[\left(b-\frac{1}{2}\right)^2 - \frac{1}{4}\right]$

$\left(b-\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4}a(a-1)$

$b = \frac{1}{2} \pm \sqrt{\frac{1}{4}+\frac{1}{4}a(a-1)}$

$= \frac{1}{2} + \frac{1}{2}\sqrt{a^2-a+1}$ or $\frac{1}{2} - \frac{1}{2}\sqrt{a^2-a+1}$ (rejected $\because b>0$)

5. Discrete Random Variables / Binomial Distribution

6R, 3Y

T = no. of counters Lee removes from bag

(i)

t	$P(T=t)$
2	$\left(\frac{6}{9}\right)\left(\frac{5}{8}\right) = \frac{5}{12}$
3	$2\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\left(\frac{5}{7}\right) = \frac{5}{14}$
4	$3\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\left(\frac{2}{7}\right)\left(\frac{5}{6}\right) = \frac{5}{28}$
5	$4\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)\left(\frac{5}{5}\right) = \frac{1}{21}$

(ii) Using GC, $E(T) = 2.86$ or $\frac{20}{7}$

$var(T) = (0.8748177165)^2 = 0.765$ (3 s.f.)

(iii) Let X be the no. of games Lee takes ≥ 4 counters out of 15 games

$X \sim B\left(15, \frac{5}{28} + \frac{1}{21}\right)$

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 0.238$

6. Permutations & Combinations / Probability

20 cards — 5 sets of 4 cards

R, B, G, Y, O   Family: F, M, D, S

(i) No. of ways 4 cards in each family are next to each other

$= 5! \times (4!)^5 = 955514880$

(ii) All 5 F together, all 4R together + all 4B together

$\underbrace{RRR\,F_R F\,FF\,F_B BBB}_{1 \text{ unit}} + 9$ others $= 10$ units

arrange 3F in middle, arrange R + B families

No. of ways $= 10! \times 3! \times 3! \times 3! \times 2! = 156764600$

arrange 3R, arrange 3B

(iii) P(no two F cards next to each other)

$= \dfrac{(15-1)! \times {}^{15}C_5 \times 5!}{(20-1)!} = \dfrac{1001}{3876}$ or 0.258

7. Hypothesis Testing

(i) For a sample to be random, every biscuit bar produced has an equal chance of being selected for the sample

(ii) Unbiased estimate of $\mu = \dfrac{\Sigma(x-32)}{n} + 32$

$= \dfrac{-7.7}{40} + 32$

$= 31.8075 \approx 31.8$ (3 s.f.)

Unbiased estimate of $\sigma^2 = \frac{1}{n-1}\left[\Sigma(x-32)^2 - \dfrac{(\Sigma(x-32))^2}{n}\right]$

$= \frac{1}{39}\left[11.05 - \dfrac{(-7.7)^2}{40}\right]$

$= 0.2453269 \approx 0.245$

(iii) Let X be the mass of biscuit bars &
μ " " population mean mass of biscuit bars.

To test $H_0 : \mu = 32$
against $H_1 : \mu \neq 32$

$\because n = 40$ is large, by Central Limit Theorem,

$\bar{X} \sim N(32, \frac{s^2}{40})$ approximately

Test Statistic, $Z = \dfrac{\bar{X} - 32}{s/\sqrt{40}} \sim N(0,1)$

Reject H_0 if $p < \alpha = 0.01$

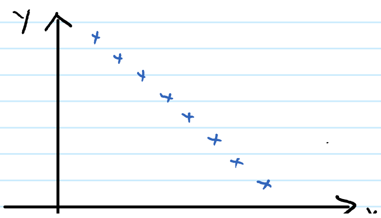
Using GC, $p = 0.013969$, $z = -2.4580$

$\because p = 0.0140 > 0.01$, we do not reject H_0 and conclude that there's insufficient evidence at 1% level of significance that mean mass not equal 32g.

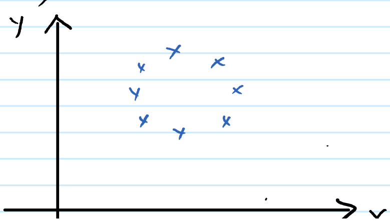
(iv) No need for product manager to know distribution of mass of biscuit bars $\because n = 40$ is large, by Central Limit Theorem, the mean mass of biscuit bars follows normal distribution approximately.

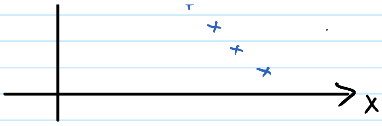
8. Correlation and Regression

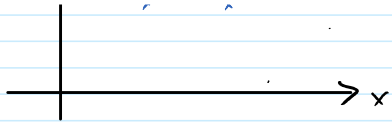
(a) (i) $r = -1$



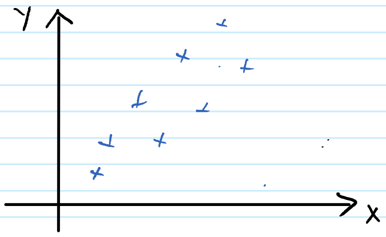
(ii) $r = 0$



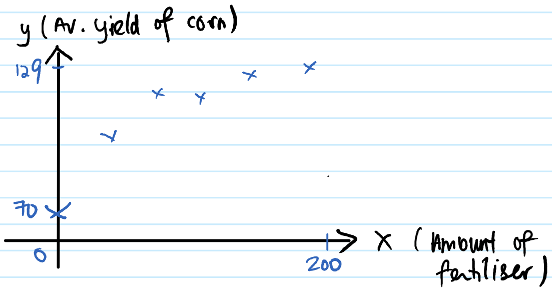




(iii) $0.5 < r < 0.9$



(b) (i)



(D) $y = a\sqrt{x} + b$ is the most accurate model

(ii) $y = 4.182114\sqrt{x} + 74.04787$

$\approx 4.18\sqrt{x} + 74.0$ (3 s.f.)

$r = 0.98085 \approx 0.981$

(iii) Reasonable to estimate $x = 189$ ∵ it lies within the given data range and $r = 0.981$ is close to 1 which indicates a strong positive linear correlation between average yield of corn and amount of fertiliser.

9. Binomial Distribution / Probability

(i) The probability that a kitchen light is faulty is constant. Whether a kitchen light is fault or not is independent of each other.

(ii) Let X be no. of faulty lights out of 12

$X \sim B(12, 0.08)$

$P(X \geq 1) = 1 - P(X = 0)$

$= 0.632336 \approx 0.632$ (3 s.f.)

(iii) P(each box in carton contains ≥ 1 faulty light)

$= (0.632336)^{20} = 1.0445 \times 10^{-4}$

$\approx 1.04 \times 10^{-4}$ (3 s.f.)

(iv) Let y be no of faulty lights out of 240

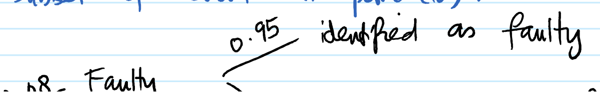
$y \sim B(240, 0.08)$

$P(y \geq 20) = 1 - P(y \leq 19)$

$= 0.45833 \approx 0.458$ (3 s.f.)

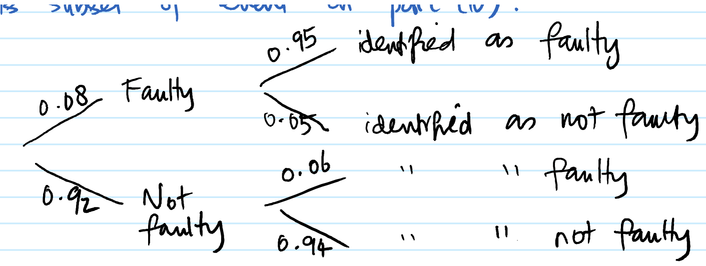
(v) Answer to part (iv) > answer to part (iii) as event in part (iii) is subset of event in part (iv).

(vi)



is subset of event in part (iv).

(vi)



P(not faulty | identified as faulty)

$= \dfrac{P(\text{not faulty} \cap \text{identified as faulty})}{P(\text{identified as faulty})} = \dfrac{0.92(0.06)}{0.08(0.95) + 0.92(0.06)}$

$= 0.42073$ or $\dfrac{69}{164}$

≈ 0.421 (3 s.f.)

(vii) P (test correctly identifies light as faulty or not faulty)

$= 0.08(0.95) + 0.92(0.94)$

$= 0.9408 \approx 0.941$

(viii) Quick test is not worthwhile as there's 42.1% of a light identified as faulty by quick test is not faulty.

10. Normal Distribution

(i) Let X be the masses of spheres

$X \sim N(20, 0.5^2)$

$P(X > 20.2) = 0.34456 \approx 0.345$ (3 s.f.)

(ii) Let Y be masses of coated spheres

$Y = 1.1X \sim N(1.1(20), 1.1^2(0.5^2))$

$P(21.5 < Y < 22.45) = 0.611722$

≈ 0.612 (3 s.f.)

(iii) Let W be masses of metal bar.

$W \sim N(\mu, \sigma^2)$

$P(W > 12.2) = 0.6$

$1 - P(W \leq 12.2) = 0.6 \Rightarrow P\left(Z \leq \dfrac{12.2 - \mu}{\sigma}\right) = 0.4$

$\dfrac{12.2 - \mu}{\sigma} = -0.2533471 \Rightarrow \mu - 0.2533471\sigma = 12.2$ —— (1)

$P(W < 12) = 0.25 \Rightarrow P\left(Z < \dfrac{12 - \mu}{\sigma}\right) = 0.25$

$\dfrac{12 - \mu}{\sigma} = -0.6744897 \Rightarrow \mu - 0.6744897\sigma = 12$ —— (2)

Using GC, $\mu = 12.303, \approx 12.3$

$\sigma = 0.4748985 \approx 0.475$

(iv) Let T be total mass of a component

$T = X_1 + X_2 + W \sim N(2(22) + 12.303, 2(0.55^2) + 0.474898^2)$

$T \sim N(56.3203, 0.830528)$

$P(T > k) = 0.75$

$1 - P(T \leq k) = 0.75 \Rightarrow P(T \leq k) = 0.25$

$\therefore \ k = 55.7056$

≈ 55.7